This exam consists of 3 numbered pages

ERASMUS UNIVERSITY ROTTERDAM Entrance examination Mathematics level 2 for International Bachelor Economics & Business Economics (IBEB) PRACTICE EXAM

Please note:

- Available time: 2.5 hours (150 minutes).
- The use of a graphing calculator or of a so called programmable calculator is not permitted. The use of a simple scientific calculator is allowed.
- Whenever possible, avoid decimal approximations and give an exact answer.
- In all your answers, give a complete solution where you show all the required steps, formulas, and substitutions that lead to your answer.
- A good or wrong answer is only a small part of the solution. The quality and completeness of your detailed solutions determine the points you will get. You should end an exercise with a conclusion or an answer.

Question 1

Solve each of the systems of equations below:

(a)
$$\begin{cases} 3x + 11y = 21 \\ 2x + 5y = 7 \end{cases}$$
 (b)
$$\begin{cases} 3x - 4y = 3 \\ \frac{1}{2}x + \frac{2}{3}y = -\frac{1}{6} \end{cases}$$

Question 2

Determine the derivative of each of the following functions, and simplify your answer as much as possible.

(a)
$$f(x) = \sqrt{x^2 + 4}$$

(b) $f(x) = \ln(x^2 - 6x) - \ln(x)$
(c) $f(x) = \frac{x^2 + 8x + 15}{4x^2 + 8x + 15}$

$$(c) \quad f(x) = \frac{1}{5+x}$$

Question 3

Given the function $f(x) = e^{-x^2 + 2x}$

- (a) Determine the extreme values of f(x) or show that f has no extreme values. Determine for each extreme value whether it is a (local) maximum or a (local) minimum.
- (b) Show that $f''(x) = 2(2x^2 4x + 1)e^{-x^2+2x}$. Furthermore, determine the points of inflection of f or show that f has no inflection points.
- (c) Determine the domain of function f and sketch the graph of f, based on the answers to the previous questions.

Question 4

Solve the following equations:

(a)
$$\ln(x^4 - 24x^2) - \ln(x^2) = 0$$

(b)
$$\sqrt{x^2 - 15x} - x = 5$$

(c)
$$(3^x)^2 = \frac{1}{9^{3x+4}}$$

Question 5

Consider the functions f(x) = |2x| and $g(x) = x^2 - 3$ Note: |x| denotes the absolute value of x.

(a) Sketch the graphs of f and g in one figure.

(b) Solve
$$|2x| = x^2 - 3$$

(c) Solve $|2x| \le x^2 - 3$

Question 6

- (a) Determine the formula of the straight line through the point (10, 3), and parallel to the line $y = \frac{2}{5}x + 13$.
- (b) Determine the formula of the straight line through the point (10, 3), and perpendicular to the line $y = \frac{2}{5}x + 13$.
- (c) Determine the formula of the tangent line of $f(x) = \frac{2}{5}x^2 + 13x 167$ at the point (10, 3).

Question 7

(a) In the (x, y)-plane, sketch the region given by:

 $y \le \ln(-x)$ and $y \le \ln(x+7)$ and $y \ge -1$

(b) Give the (x, y)-coordinates of the three vertices of this region.

Question 8

- (a) Solve $3x^2 4x 4 = 0$
- (b) Solve $3x^2 + 12x + 12 = 0$
- (c) Give all value(s) of p for which the equation $3x^2 + px + p = 0$ has no solutions.

Question 9

Consider the function $f(x) = ax^4 - 8x^3 + b$. Assume that (x, y) = (2, 8) is an inflection point of this function. Show that f has another inflection point and compute the (x, y)-coordinates of this other point of inflection.