## ERASMUS UNIVERSITY ROTTERDAM
### Entrance examination Mathematics level 3 for Econometrics

## SAMPLE QUESTIONS

### General information concerning the entrance exam

**A: The following information will be printed on the title page of your entry test:**

- **Available time: 3 hours (180 minutes).**

- **The use of a calculator is not permitted.**

- **In all your answers, give a complete solution where you show all the required steps, formulas, and substitutions that lead to your answer.**

- **A good or wrong answer is only a small part of the solution. The quality and completeness of your detailed solutions determine the points you will get. You should end an exercise with a conclusion or an answer.**

**B: Typically, the exam will start with two or three general questions concerning (systems of) equations, derivatives and/or integrals:**

### Question 1

Solve the following equations:

(a) $\dfrac{6x^2 - 12}{(x^2 - 1)^2} = \dfrac{4}{3}$

(b) $e^{4x} - 3e^{x+1} = 0$

(c) $^3\log(x + 2) = 1 - {}^3\log(x)$

### Question 2

Determine the derivative of each of the following functions:

(a) $f(x) = (x^5 - 3x)^3 \sin x$

(b) $f(x) = \ln(2x + \cos(3x))$

(c) $f(x) = \dfrac{x^3 \ln x}{x^2 - 3}$

**Question 3**

Calculate the following integrals:

(a) $\displaystyle\int \frac{x}{4-x^2}\,dx$

(b) $\displaystyle\int_{\frac{1}{2}}^{3\frac{1}{2}} \frac{4}{\sqrt{4x+2}}\,e^{\sqrt{4x+2}}\,dx$

(c) $\displaystyle\int_0^1 f(x)\,dx$ under the following conditions: $f'(x) = \dfrac{24}{(4-2x)^4}$ and $f(0) = 1$

**C: the remainder of the exam will consist of three to five more elaborate problems. These problems, or subquestions of these problems, may resemble the following sample questions:**

**Question 4**

Consider the functions $f(x) = x^3$ and $g(x) = x\sqrt{x} + 2$. For which values of $x$ is $f(x) < g(x)$?

**Question 5**

For every $p \in \mathbb{R}$ consider the function $f_p(x) = e^{4x - x^2 + px^3}$

(a)  Assume $p = 0$. Calculate the $x$-coordinates of the inflection points of the graph of $f_0(x)$.

(b)  Determine all values of $p$ for which the function $f_p(x)$ has exactly two extremes.

**Question 6**

Consider the functions $f(x) = 2\cos^2(x)$ and $g(x) = \sin(2x) - 1$, both with domain $[0, \pi]$.
The line $x = p$ intersects the graph of $f$ in point $A$ and the graph of $g$ in point $B$.
Calculate the value of $p$ for which the length of segment $AB$ is maximal.

**Question 7**

Calculate the exact value of $p$ for which the graphs of the functions $f(x) = 2\sqrt{x}$ and $g_p(x) = \frac{p}{x}$ intersect perpendicularly. Also, give the coordinates of the intersection point.

**Question 8**

Consider the functions $f(x) = {}^2\log(x+3)$ and $g(x) = 1 + \frac{1}{2}\log(x)$. The line $y = p$ intersects the graph of $f$ in point $A$ and the graph of $g$ in point $B$. Give all the values of $p$ for which the section $AB$ is equal to 2.

**Question 9**

Calculate the area of the region enclosed by the graphs of $f(x) = x^2$, $g(x) = \sqrt{x}$ and $h(x) = 6 - x$.

**Question 10**

Consider the function $f(x) = x \cdot e^{-x^3}$
The region bounded by $f(x)$, the $x$-axis and the line $x = 1$ is denoted by $R$.
Compute the volume of the solid generated by revolving region $R$ about the x-axis.