ERASMUS UNIVERSITY ROTTERDAM
Information concerning the Entrance examination Mathematics level 2 for International Bachelor Economics & Business Economics (IBEB)

General information

The following information will be printed on the title page of your entry test:

- Available time: 2.5 hours (150 minutes).
- The use of a graphing calculator or of a so called programmable calculator is not permitted. The use of a simple scientific calculator is allowed.
- Whenever possible, avoid decimal approximations and give an exact answer.
- In all your answers, give a complete solution where you show all the required steps, formulas, and substitutions that lead to your answer.
- A good or wrong answer is only a small part of the solution. The quality and completeness of your detailed solutions determine the points you will get. You should end an exercise with a conclusion or an answer.

Content information

In the exam you may find questions regarding the following topics:

A: NUMBERS AND RULES OF CALCULATION

1. The sets $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$ and the operations addition, subtraction, multiplication and division.

   Natural numbers: $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$
   Integers: $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$
   Rational numbers: $\mathbb{Q} = \{\frac{a}{b} \mid a \in \mathbb{Z}, b \in \mathbb{N}, b \neq 0\}$

2. Absolute value $|x|$ and simple graphs corresponding with absolute value.

$$|x| = \begin{cases} x & \textit{when} \quad x \geq 0 \\ -x & \textit{when} \quad x < 0 \end{cases}$$

B: ALGEBRAIC SKILLS

1. Fractions
   - $\frac{1}{a}+\frac{1}{b}=\frac{b}{a\cdot b}+\frac{a}{a\cdot b}=\frac{a+b}{a\cdot b}$
   - $\frac{a}{b}+c=\frac{a}{b}+\frac{b\cdot c}{b}=\frac{a+b\cdot c}{b}$
   - $\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d}{b\cdot d}+\frac{b\cdot c}{b\cdot d}=\frac{a\cdot d+b\cdot c}{b\cdot d}$
   - $a\cdot\frac{b}{c}=\frac{a\cdot b}{c}=\frac{a}{c}\cdot b=a\cdot b\cdot\frac{1}{c}$
   - $\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}$
   - $\frac{a}{\frac{b}{c}}=a\cdot\frac{c}{b}=\frac{a\cdot c}{b}$

2. Special products
   - $(a\pm b)^2=a^2\pm 2ab+b^2$
   - $(a+b)(a-b)=a^2-b^2$

3. Factorization of a polynomial of second degree (quadratic function) into linear terms.

   For example:
   - $f(x)=x^2-2x-15$ *can be factorized as follows:* $f(x)=(x-5)(x+3)$
   - $f(x)=x^2-2x$ *can be factorized as follows:* $f(x)=x(x-2)$

4. The power rules and corresponding rules for logarithms. Use of rational and negative exponents.
   - $a^p\cdot a^q=a^{p+q}$
   - $\frac{a^p}{a^q}=a^{p-q}$
   - $(a^p)^q=a^{pq}$
   - $(ab)^p=a^pb^p$
   - $\left(\frac{a}{b}\right)^p=\frac{a^p}{b^p}$
   - $a^{-p}=\frac{1}{a^p}$
   - $a^{\frac{1}{p}}=\sqrt[p]{a}$
   - ${}^g\!\log a+{}^g\!\log b={}^g\!\log ab$
   - ${}^g\!\log a-{}^g\!\log b={}^g\!\log\frac{a}{b}$
   - ${}^g\!\log a^p=p\cdot{}^g\!\log a$
   - ${}^g\!\log a=\frac{{}^p\!\log a}{{}^p\!\log g}=\frac{\ln a}{\ln g}$

   Remark: An alternative notation for ${}^g\!\log a$ *is* $\log_g a$*. The first notation will be used in the problems of the exam. Both notations are allowed in your calculations.*

5. Application of the items mentioned in B1 through 4
   - *through substitution of numbers*
   - *through substitution of expressions*
   - *by reformulation and reduction of expressions*

C: STANDARD FUNCTIONS

1. Properties of the following standard functions:
   - polynomials

     In particular:
     the linear function $y = ax + b$
     the quadratic function $y = ax^2 + bx + c$

   - the rational function $y = \frac{ax+b}{cx+d}$
   - power functions $y = c \cdot x^n$

     In particular the function $y = \sqrt{x}$

   - exponential functions $y = g^x$ and their inverse functions $y = {}^g\log x$
   - the function $y = e^x$ and its inverse function $y = \ln x$
2. Sketch the graphs of the standard functions and use the notions domain (all possible values of x for which the function is defined), range (the set of all values of y obtained as x varies in the domain), zeroes (solutions of the equation $f(x) = 0$), increasing, decreasing function and asymptotic behaviour.

   For example: The function $y = 2^x$ is increasing and has a horizontal asymptote $y = 0$; the domain of function $y = \ln x$ is equal to $(0, \infty]$, the function is increasing and has a vertical asymptote $x = 0$.

3. Apply transformations on graphs such as shifting and stretching, and describe the link between the transformation and the alteration of the graph of the corresponding function.
   - translation of $f(x)$ along the vector (a, b) results in $y = f(x - a) + b$
   - multiplication of $f(x)$ with a with respect to the x-axis results in $y = a \cdot f(x)$
   - multiplication of $f(x)$ with b with respect to the y-axis results in $y = f(x/b)$

   For example: argue that the domain of $y = \sqrt{3x - 6}$ is equal to $x \geq 2$.

   For example: sketch the graph of $f(x) = 2 + 3\sqrt{x - 6}$, where $x \geq 6$, based on a sketch of the graph of $\sqrt{x}$

4. Sketch a region bounded by the graphs of several standard functions, or by the graphs of functions obtained from standard functions after one or more transformations as described in C3. Sketches of regions need not be to scale. Points are rewarded for
   - the approximate location of asymptotes and other typical properties of each function (equivalent to item C2)
   - the approximate position of all points of intersection
   - a clearly indicated (i.e. shaded or coloured) region
   - clearly indicated vertices of the region. A vertex of a region is a point of the region, where two of its bounding graphs intersect.

   Note: when a sketch of a region is asked, points are rewarded for the sketch only. Additional calculations are not rewarded extra points.

D: EQUATIONS AND INEQUALITIES

1. Solving equations using the following general rules

   - $a \cdot b = 0 \Leftrightarrow a = 0$ *or* $b = 0$
   - $a \cdot b = a \cdot c \Leftrightarrow a = 0$ *or* $b = c$
   - $\dfrac{a}{b} = c \Leftrightarrow a = b \cdot c$ *and* $b \neq 0$
   - $\dfrac{a}{b} = \dfrac{c}{d} \Leftrightarrow a \cdot d = b \cdot c$ *and* $b, d \neq 0$
   - $a^2 = b^2 \Leftrightarrow a = b$ *or* $a = -b$

2. Solve equations with polynomials using standard algorithms

   - *Linear equations*
   - *Quadratic equations of the form* $ax^2 + b = 0$ *or* $ax^2 + bx = 0$
   - *Solve general quadratic equations of the form* $ax^2 + bx + c = 0$ *by means of factorization or by means of the so-called "quadratic formula":*

     The solutions of $ax^2 + bx + c = 0$ *are given by* $x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

   - *Linear or quadratic equations with parameters*

3. Solve equations of the type:

   $x^a = b \quad \Rightarrow \quad x = b^{\frac{1}{a}} = \sqrt[a]{b}$

   $g^x = a \quad \Rightarrow \quad x = {}^g\log a = \frac{\ln a}{\ln g}$

   ${}^g\log x = a \quad \Rightarrow \quad x = g^a$

4. Solve equations of the type $f(x) = g(x)$ algebraically, where $f(x)$ and $g(x)$ are standard functions (as described in item C1) or transformations of standard functions (as described in item C3).
5. Solve equations of the type $|f(x)| = g(x)$ algebraically, where $f(x)$ and $g(x)$ are standard functions (as described in item C1) or transformations of standard functions (as described in item C3).
6. Solve inequalities of the type $f(x) \geq g(x)$ by first solving $f(x) = g(x)$ algebraically and subsequently using a sketch of both graphs.

   For example: When solving $x^2 - 3x - 10 > -x + 5$*, we first compute the solutions of* $x^2 - 3x - 10 = -x + 5$*, i.e.* $x = 5$ *of* $x = -3$*. A sketch of the graphs supplies the answer to the problem:* $x < -3$ *or* $x > 5$*.*

E: CALCULUS

1. Recognize and use the different symbols for the derivative of a function.

   $f'(x)$, $\frac{dy}{dx}$, $\frac{df(x)}{dx}$ *and* $\frac{df}{dx}(x)$ *all denote the same notion.*

2. Compute the derivative of the sum, the product and the quotient of standard functions (as described in item C1) or of transformations of standard functions (as described in item C3). Use the chain rule to compute the derivative of combinations of these functions.

Some examples:

sum:	$f(x) = x^2 + \sqrt{x}$	$\Rightarrow$	$f'(x) = 2x + \dfrac{1}{2\sqrt{x}}$
product:	$f(x) = (x^2 - 3x + 5)\ln x$	$\Rightarrow$	$f'(x) = (2x - 3)\ln x + (x^2 - 3x + 5) \cdot \dfrac{1}{x}$
quotient:	$f(x) = \dfrac{x^2}{5x+3}$	$\Rightarrow$	$f'(x) = \dfrac{(5x+3)\cdot 2x - x^2(5)}{(5x+3)^2} = \dfrac{5x^2+6x}{(5x+3)^2}$
chain rule:	$f(x) = e^{4x^2-5}$	$\Rightarrow$	$f'(x) = 8xe^{4x^2-5}$

3. Use the differential quotient $(\Delta y/\Delta x)$ as indication of the local change of a function.
4. Determine on which intervals a function is constant (derivative $= 0$), increasing (derivative > 0) or decreasing (derivative < 0).
5. Determine (by inspecting the sign of the derivative) whether a function has maximum and/or minimum values, and calculate the coordinates of these extreme values.

   Example: the derivative of the function $f(x) = x^2 - 2x - 15$ is equal to 0 when $x = 1$. For every $x < 1$ the function is decreasing, for every $x > 1$ the function increasing, so the function has a minimum at $x = 1$. The coordinates of this minimum are $(1, -16)$.

6. Determine the second derivative $f''(x)$ of the functions described in item E2.
7. Determine on which intervals a function is convex (second derivative ≥ 0) or concave (second derivative ≤ 0).

   The second derivative of the function $f(x) = (x-1)^3 + 4$ is equal to $f''(x) = 6(x-1)$. This second derivative is equal to 0 only if $x = 1$. For every $x \leq 1$ the function f is concave and for every $x \geq 1$ the function is convex.

   Remark: Alternative names for ‘convex’ are ‘concave upward’ and ‘convex downward’. Alternative names for ‘concave’ are ‘concave downward’ and ‘convex upward’. Only the names ‘convex’ and ‘concave’ will be used in the exam.
8. Determine (by inspecting the sign of the second derivative) whether a function has points of inflection, and calculate the coordinates of such inflection points.

   - *The function $f(x) = (x-1)^3 + 4$ in the example above has a point of inflection at $x = 1$. The coordinates are $(1, 4)$.*
   - *The second derivative of the function $f(x) = (x-1)^4 + 4$ is equal to $f''(x) = 12(x-1)^2$. This second derivative is equal to 0 only if $x = 1$. For every $x < 1$ the function f satisfies $f''(x) > 0$ and for every $x > 1$ the function also satisfies $f''(x) > 0$. Hence, this function does not have any point of inflection.*

9. Sketch the graphs of non-standard functions. Sketches of graphs need not be to scale. Points are rewarded for
   - a clearly indicated domain, in case the domain is not equal to $\mathbb{R}$.
   - the approximate location of asymptotes and other typical properties of the function (equivalent to item C2)
   - correctly indicated items (such as zeroes, extremes, inflection points, convexity) but only when specifically requested.
   - in case of multiple functions in one graph: the approximate position of all points of intersection.

   When a sketch of a graph is asked, points are rewarded for the sketch only. Additional calculations are not rewarded extra points. Note: unless specifically requested, extremes and inflection points need not be calculated.

F: STRAIGHT LINES AND SYSTEMS OF EQUATIONS

1. Determine the formula of a straight line either in case two of its points are given, or in case a single point and the slope of the line are given.
2. Know the conditions for two parallel straight lines and two perpendicular straight lines.

   The two lines $y = ax + b$ and $y = cx + d$ are parallel when $a = c$. The lines are perpendicular when $ac = -1$.

3. Give the formulas for the tangent line and the normal line (i.e. the line perpendicular to the tangent line) at a given point on the graph of (combinations of) standard functions.

   Example: the function $f(x) = x^2 - 2x - 15$ satisfies $f(6) = 9$ and $f'(6) = 10$. The tangent line of the graph of $f(x)$ at $x = 6$ is described by $y = 10x - 51$. The normal line at the point with x-coordinate equal to 6 is given by $y = -\frac{1}{10}x + \frac{96}{10}$.

4. Solve a system of two linear equations with two unknowns.

   For example:

   $$\begin{cases} 2x + 2y = 2 \\ 3x + y = 5 \end{cases} \quad \Rightarrow \quad \begin{cases} x = 2 \\ y = -1 \end{cases}$$

5. Sketch the solution of a system with linear inequalities.