This exam consists of
3 numbered pages

ERASMUS UNIVERSITY ROTTERDAM

Colloquium doctum Mathematics level 2

for International Bachelor Economics & Business Economics (IBEB)
and for International Business Administration (IBA)

PRACTICE EXAM

Please note:

- Write your student number on each sheet of the exam (this number is mentioned in the letter you have received from the exam administration)
- The use of a so called "graphing calculator" or "programmable calculator" is not permitted. "Simple" scientific calculators are allowed.
- Available time: 2.5 hours (150 minutes). You are not allowed to leave the examination room during the first hour of the exam and during the last 15 minutes.
- You CANNOT take any part of the exam home. All questions and answers have to be returned to the supervisor.
- In all your answers, give a complete solution where you show all the required steps, formulas, and substitutions that lead to your answer. A good or wrong answer is only a small part of the solution. The quality and completeness of your detailed solutions determine the points you will get. You should end an exercise with a conclusion or an answer.
- Questions containing the words "solve", "derive" or "calculate" require an exact answer; a decimal approximation is not allowed.

Question 1

Solve each of the systems of equations below:

(a) $\begin{cases} 3x + 11y &= 21 \\ 2x + 5y &= 7 \end{cases}$ (b) $\begin{cases} 5x - 2y &= -7 \\ 11x - 3y &= -21 \end{cases}$

Question 2

Determine the derivative of each of the following functions, and simplify your answer as much as possible.

(a) $f(x) = \sqrt{x^2 + 4}$

(b) $f(x) = \ln(x^2 - 6x) - \ln(x)$

(c) $f(x) = \dfrac{x^2 + 8x + 15}{5 + x}$

Question 3

Given the function $f(x) = e^{-x^2+2x}$

(a) Determine the extreme values of $f(x)$ or show that f has no extreme values. Determine for each extreme value whether it is a (local) maximum or a (local) minimum.

(b) Show that $f''(x) = 2(2x^2 - 4x + 1)e^{-x^2+2x}$. Furthermore, determine the points of inflection of f or show that f has no inflection points.

(c) Determine the domain of function f and sketch the graph of f, based on the answers to the previous questions.

Question 4

Solve the following equations:

(a) $\ln(x^4 - 24x^2) - \ln(x^2) = 0$

(b) $\sqrt{x^2 - 15x} - x = 5$

(c) $(3^x)^2 = \dfrac{1}{9^{3x+4}}$

Question 5

Consider the functions $f(x) = |\,2x\,|$ and $g(x) = x^2 - 3$
Note: $|\,x\,|$ denotes the absolute value of x.

(a) Sketch the graphs of f and g in one figure.

(b) Solve $|\,2x\,| = x^2 - 3$

(c) Solve $|\,2x\,| \leq x^2 - 3$

Question 6

(a) Determine the formula of the straight line through the point $(10, 3)$, and parallel to the line $y = \frac{2}{5}x + 13$.

(b) Determine the formula of the straight line through the point $(10, 3)$, and perpendicular to the line $y = \frac{2}{5}x + 13$.

(c) Determine the formula of the tangent line of $f(x) = \frac{2}{5}x^2 + 13x - 167$ at the point $(10, 3)$.

Question 7

(a) Sketch in the same figure the graphs of the functions

$$f(x) = \frac{x + \frac{1}{2}}{2} \quad \text{and} \quad g(x) = \frac{2}{x + \frac{1}{2}}$$

(b) Calculate all points of intersection of the graphs of f and g.

(c) Determine all the values of x for which the inequality $f(x) \geq g(x)$ holds.

Question 8

(a) For each of the following three equations, give the number of solutions (explain your answer).

(i) $3x^2 - 2x - 2 = 0$
(ii) $3x^2 + 8x + 8 = 0$
(iii) $3x^2 + 20x + 20 = 0$

(b) Give all value(s) of p for which the equation $3x^2 + px + p = 0$ has no solutions.

Question 9

Consider the function $f(x) = ax^4 - 8x^3 + b$. Assume that $(x, y) = (2, 8)$ is an inflection point of this function. Show that f has another inflection point and compute the (x, y)-coordinates of this other point of inflection.