ERASMUS UNIVERSITY ROTTERDAM
Entrance examination Mathematics level 3 for Econometrics

SAMPLE QUESTIONS

General information concerning the entrance exam

A: The following information will be printed on the title page of your entry test:

- Available time: 3 hours (180 minutes).
- The use of a calculator is not permitted.
- In all your answers, give a complete solution where you show all the required steps, formulas, and substitutions that lead to your answer.
- A good or wrong answer is only a small part of the solution. The quality and completeness of your detailed solutions determine the points you will get. You should end an exercise with a conclusion or an answer.

B: Typically, the exam will start with two or three general questions concerning (systems of) equations, derivatives and/or integrals:

Question 1

Solve the following equations:

(a) \( \frac{6x^2 - 12}{(x^2 - 1)^2} = \frac{4}{3} \)
(b) \( e^{4x} - 3e^{x+1} = 0 \)
(c) \( 3\log(x + 2) = 1 - 3\log(x) \)

Question 2

Determine the derivative of each of the following functions:

(a) \( f(x) = (x^5 - 3x)^3 \sin x \)
(b) \( f(x) = \ln(2x + \cos(3x)) \)
(c) \( f(x) = \frac{x^3 \ln x}{x^2 - 3} \)
Question 3

Calculate the following integrals:

(a) \[ \int \frac{x}{4-x^2} \, dx \]

(b) \[ \int_{\frac{1}{2}}^{3} \frac{4}{\sqrt{4x+2}} \, e^{\sqrt{4x+2}} \, dx \]

(c) \[ \int_{0}^{1} f(x) \, dx \] under the following conditions: \( f'(x) = \frac{24}{(4-2x)^4} \) and \( f(0) = 1 \)

C: the remainder of the exam will consist of three to five more elaborate problems. These problems, or subquestions of these problems, may resemble the following sample questions:

Question 4

Consider the functions \( f(x) = x^3 \) and \( g(x) = x\sqrt{x} + 2 \). For which values of \( x \) is \( f(x) < g(x) \)?

Question 5

For every \( p \in \mathbb{R} \) consider the function \( f_p(x) = e^{4x-x^2+px^3} \)

(a) Assume \( p = 0 \). Calculate the \( x \)-coordinates of the inflection points of the graph of \( f_0(x) \).

(b) Determine all values of \( p \) for which the function \( f_p(x) \) has exactly two extremes.

Question 6

Consider the functions \( f(x) = 2\cos^2(x) \) and \( g(x) = \sin(2x) - 1 \), both with domain \([0, \pi]\). The line \( x = p \) intersects the graph of \( f \) in point \( A \) and the graph of \( g \) in point \( B \). Calculate the value of \( p \) for which the length of segment \( AB \) is maximal.
Question 7

Calculate the exact value of $p$ for which the graphs of the functions $f(x) = 2\sqrt{x}$ and $g_p(x) = \frac{p}{x}$ intersect perpendicularly. Also, give the coordinates of the intersection point.

Question 8

Consider the functions $f(x) = 2\log(x + 3)$ and $g(x) = 1 + \frac{1}{2}\log(x)$. The line $y = p$ intersects the graph of $f$ in point $A$ and the graph of $g$ in point $B$. Give all the values of $p$ for which the section $AB$ is equal to 2.

Question 9

Calculate the area of the region enclosed by the graphs of $f(x) = x^2$, $g(x) = \sqrt{x}$ and $h(x) = 6 - x$.

Question 10

Consider the function $f(x) = x \cdot e^{-x^3}$
The region bounded by $f(x)$, the $x$-axis and the line $x = 1$ is denoted by $R$.
Compute the volume of the solid generated by revolving region $R$ about the x-axis.