This exam consists of
3 numbered pages

ERASMUS UNIVERSITY ROTTERDAM
Entrance examination Mathematics level 2
for International Bachelor Economics & Business Economics (IBEB)
PRACTICE EXAM

Please note:
• Available time: 2.5 hours (150 minutes).

• In all your answers, give a complete solution where you show all the required steps, formulas, and substitutions that lead to your answer. A good or wrong answer is only a small part of the solution. The quality and completeness of your detailed solutions determine the points you will get. You should end an exercise with a conclusion or an answer.

• The use of a graphing calculator or of a so called programmable calculator is not permitted. The use of a simple scientific calculator is allowed.
Question 1

Solve each of the systems of equations below:

(a) \[
\begin{align*}
3x + 11y &= 21 \\
2x + 5y &= 7
\end{align*}
\]

(b) \[
\begin{align*}
5x - 2y &= -7 \\
11x - 3y &= -21
\end{align*}
\]

Question 2

Determine the derivative of each of the following functions, and simplify your answer as much as possible.

(a) \[f(x) = \sqrt{x^2 + 4}\]

(b) \[f(x) = \ln(x^2 - 6x) - \ln(x)\]

(c) \[f(x) = \frac{x^2 + 8x + 15}{5 + x}\]

Question 3

Given the function \(f(x) = e^{-x^2+2x}\)

(a) Determine the extreme values of \(f(x)\) or show that \(f\) has no extreme values. Determine for each extreme value whether it is a (local) maximum or a (local) minimum.

(b) Show that \(f''(x) = 2(2x^2 - 4x + 1)e^{-x^2+2x}\). Furthermore, determine the points of inflection of \(f\) or show that \(f\) has no inflection points.

(c) Determine the domain of function \(f\) and sketch the graph of \(f\), based on the answers to the previous questions.

Question 4

Solve the following equations:

(a) \[\ln(x^4 - 24x^2) - \ln(x^2) = 0\]

(b) \[\sqrt{x^2 - 15x} = x - 5\]

(c) \[(3^x)^2 = \frac{1}{9^{3x+4}}\]
Question 5
Consider the functions \( f(x) = |2x| \) and \( g(x) = x^2 - 3 \)
Note: \( |x| \) denotes the absolute value of \( x \).
(a) Sketch the graphs of \( f \) and \( g \) in one figure.
(b) Solve \( |2x| = x^2 - 3 \)
(c) Solve \( |2x| \leq x^2 - 3 \)

Question 6
(a) Determine the formula of the straight line through the point \((10, 3)\), and parallel to the line \( y = \frac{2}{5}x + 13 \).
(b) Determine the formula of the straight line through the point \((10, 3)\), and perpendicular to the line \( y = \frac{2}{5}x + 13 \).
(c) Determine the formula of the tangent line of \( f(x) = \frac{2}{5}x^2 + 13x - 167 \) at the point \((10, 3)\).

Question 7
(a) Sketch in the same figure the graphs of the functions
\[
f(x) = \frac{x + \frac{1}{2}}{2} \quad \text{and} \quad g(x) = \frac{2}{x + \frac{1}{2}}
\]
(b) Calculate all points of intersection of the graphs of \( f \) and \( g \).
(c) Determine all the values of \( x \) for which the inequality \( f(x) \geq g(x) \) holds.

Question 8
(a) For each of the following three equations, give the number of solutions (explain your answer).
   (i) \( 3x^2 - 2x - 2 = 0 \)
   (ii) \( 3x^2 + 8x + 8 = 0 \)
   (iii) \( 3x^2 + 20x + 20 = 0 \)
(b) Give all value(s) of \( p \) for which the equation \( 3x^2 + px + p = 0 \) has no solutions.

Question 9
Consider the function \( f(x) = ax^4 - 8x^3 + b \). Assume that \((x, y) = (2, 8)\) is an inflection point of this function. Show that \( f \) has another inflection point and compute the \((x, y)\)-coordinates of this other point of inflection.