Erasmus School of Economics

Report on the use of math and game theory in the bachelor programme Economics and Business Economics

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Summary and Recommendations

In May 2016, students in the Programme Committee Economics and Business Economics (PC) initiated a discussion on the use of math and game theory in the bachelor programme. Key concerns were: a) the level of math, b) a potential mismatch between what is taught in bachelor-1 and what is used later, and c) repeated discussion of the same techniques and concepts in different courses.
As we had only partial information about the use of math in the programme, we decided to gather information among students and lecturers, using focus groups and surveys. This report is the product of these efforts.

The focus group sessions conducted with bachelor-3 students largely confirmed the initial concerns. Regarding the level of math, some students considered it to be adequate while others would have preferred a higher level. Students in nearly all majors confirmed that many skills taught in bachelor-1 are not used in bachelor-2. Depending on the major, the use of some of these skills returns in bachelor-3. Students asked for more opportunities to practice these skills. Lastly, students felt that sometimes they had to learn tricks without understanding why.

The survey held among lecturers of all bachelor-1 and 2 courses showed that the skills taught in Math I (mainly algebra, differentiation, and optimization) are used in various courses. However, the skills taught in Math II (mainly integration and matrix algebra) are essentially not used. Regarding game theory, we found that the basic concepts (simple games of complete information, Nash-equilibrium, subgame perfection) are applied in various courses.

The survey held among lecturers of all courses included in any major offered in bachelor-3 showed that all skills taught are used in multiple majors. There are no skills used in in only one or two majors.

Lastly, both lecturers and students widely share the observation that the heterogeneity in student ability regarding math and game theory is large. This holds when students enter our bachelor programme, but also among students in the various major programmes in bachelor-3.

We use these findings to provide a number of recommendations aimed at improving the use of math and game theory (M&GT). Below, we list these recommendations, grouped into in three dimensions: (i) information sharing, (ii) coordination, and (iii) programme structure.

(i) Information sharing
   a. Make this report available to all lecturers and students.
   b. Give each lecturer reading access to the learning environment (blackboard / canvas) of all courses, at least of the courses in bachelor-1 and bachelor-2.
   c. In teaching skills to students, explain why they need to learn these skills:
      i. Mention in which courses the skills will be used, and why.
      ii. When teaching / applying skills, also focus on the logic behind it.

(ii) Coordination:
   a. Create web-casts on the most-used M&GT skills and concepts, and make these available to all students and lecturers.
   b. Coordinate how game theory is taught, and use the same approach, supported by webcasts.

c. Coordinate an increase in the use of the math skills taught in Math II (integrals, matrix algebra, limits) in bachelor-2.
d. Integrate some Economics into the math courses, to show students why these skills are useful.

(iii) Programme structure:
a. Do not change the content of Math I.
b. Skills currently taught in Math II that cannot be integrated into bachelor-2 courses (see (ii) c), should not be taught in Math II. Instead, the teaching of these skills should move either to the second part of bachelor-2 or to (a subset of) the majors.
c. Consider introducing an optional advanced math course in bachelor-3.
d. Consider introducing an optional more advanced math course for a sub-set of students in bachelor-1.
e. Consider dedicating a part of a course in bachelor-1 (or early bachelor-2) to game theory.

We are aware that M&GT is only a subset of skills taught and used. To keep this project manageable, we decided against including other skills, in particular those related to statistics and econometrics / empirical modelling. Clearly, the importance of M&GT relative to empirical modelling differs across fields / majors. Our recommendations should not be interpreted as a call to increase the use of M&GT at the expense of statistics and empirical modelling.

Interestingly, even though this was not our focus, both lecturers of some programs as well as the students in multiple majors mentioned a (mis)match between the expected level of statistics and empirical models in year 3 and the statistics we teach in the years before (mostly in the courses Applied Statistics and Methods & Techniques). In particular, students seem to miss a link between the theory and how and when to use certain methods, echoing the complaint of “learning tricks” regarding M&GT. This may need to be verified in a follow-up of the current exercise. However, it may be worth it to start working towards a reduction of this mismatch.
Hence, we also offer two recommendation regarding statistics and empirical modelling:

(iv) Statistics and empirical modelling
a. Conduct a similar exercise on the use of statistics and empirical modelling
b. When teaching and using statistics and empirical modelling, also (briefly) focus on the math behind, and the economic use for, the statistics and empirical models.

Introduction

In May 2016, the students in the Program Committee (PC) Economics & Business Economics initiated a discussion on the use of math and game theory in the bachelor program. They listed the following concerns:

- The level of math and game theory in the bachelor does not prepare them sufficiently for several master specializations.
- Some math skills taught in the math courses are not used in later courses, or only at the end of bachelor 3.
- Some math and game theory skills are taught several times, and it seems that lecturers are not always aware of this.

In the discussion that followed, it was clear that everyone only had a partial view on the use of math and game theory skills across the bachelor. In order to be able to assess whether and where there is room for improvement, information on the use of these skills is necessary. Therefore, the PC decided to map the current use of math and game theory in the bachelor, as well as the preferences and experiences of lecturers and students.

Focus on Math and Game Theory

The PC decided to keep this exercise focused on math and game theory. We discussed broadening it to other areas (in particular to statistics and econometrics). However, we felt this would make the project too big to manage. Our results and recommendations should be seen with this decision in mind. Math is fundamental to all areas in Economics and Business Economics, but its use might differ per major/master. Furthermore, game theory is used extensively in some majors but not in others. Inevitably, this leads to different preferences over which skills should receive more attention. Hence, given our broad bachelor programme, this exercise is not aimed at increasing the use of M&GT at the expense of other skills, most notably empirical modelling. Our recommendations should be interpreted with this in mind.

Interestingly, even though it was not our focus, lecturers in several majors as well as students of multiple tracks mentioned statistics and empirical models as area's for improvement. This suggests that there also some degree of mismatch between what is taught regarding statistics and empirical models early on in the programme and what is expected and used later in the programme. This might need to be verified in a follow-up of the current exercise. However, we do report these concerns, hoping that this allows for improvements in the short run.

The relevance of math and game theory for Economics and Business Economics

One of the distinctive features of research in economics and business economics in relation with other social sciences is the use of formal theories. Mathematical models are used in the development of theory as well as in the development of empirical specifications. At the end of our bachelor programme, students should be able to independently understand academic economics articles. Hence, this requires that students receive sufficient training in these skills.

Game theory offers a framework to analyse situations of strategic interaction. In such situations, the optimal behavior of one actor depends on the behavior of another actor. As this applies to many real-world settings, the use of game theory in economics and business economics has increased substantially in the past decades. It is now applied in many fields, from marketing to international trade, and from political economy to accounting.

Besides the relevance of math and game theory for understanding academic articles, math and game theory benefit students in at least three more ways. First, math and (in particular) game theory, train students in structured reasoning. Second, as students in economics and business economics have always received more training in quantitative skills than students in other social sciences, (prospective) employers expect our graduates to have relatively strong analytical and quantitative skills. Lastly, master programmes in Economics and Business Economics abroad expect a certain level of quantitative skills. If we want our students to thrive in these programmes, they should be prepared well.

Goals

We hope that mapping the current and preferred use of math and game theory in the bachelor can lead to improvement along three dimensions:

1. Information sharing:
   Lecturers earlier in the programme can see when particular skills will be used in later courses and can refer to this during the course. This increases relevance for students. Lecturers later in the programme can see which skills students are supposed to possess, and which skills need more attention.

2. Coordination:
   a. When particular skills are taught or used in several courses followed by the same students, coordination on when and how to teach these skills can be useful.
   b. We highlight mismatches in current supply of and demand for math and game theory. We can detect which skills are taught but hardly used after. Similarly, we detect which skills are not specifically taught, even though they are used in many courses.

3. Structure:
   a. We highlight which skills are mostly used within one major specialization and which skills are used more broadly. This can inform the discussion of when a particular skill should be taught.
   b. We can detect whether there are skills that extra training in bachelor-2 to make students well-prepared for their use in bachelor-3.

Setup

In December 2016, we met with the lecturers courses Math I, Math II, Microeconomics, and Statistics I. In these courses, math and, to a smaller extent, game theory are initially taught. This gave us a list of skills taught early in the programme. Based on this list of skills, in Spring 2017 we designed a survey for lecturers of courses in block 3 and 4 of bachelor-3 and a similar survey for lecturers of the remaining courses in bachelor-1 and bachelor-2. Furthermore, we held focus group sessions with several groups of students in different seminars in bachelor-3.

M&GT skills in courses Math I, Math II, and Microeconomics

In the bachelor programme, students start with a number of key courses for M&GT skills:

- Math I, taught in block 1
- Microeconomics, block 2
- Math II, block 3

Math I trains the following skills:

- Use of different types of functions (e.g. linear, log, exponential)
- Solving equations, including linear systems with multiple unknowns
- Derivatives and rules of differentiation, for different types of functions
- Derivatives of implicit functions, including chain rule
- Limits (also partially in Math II)
- Optimization and optimization under constraints (substitution method and Lagrange method)
- Limited attention for comparative statics, concavity, and convexity.

Microeconomics uses / trains the following skills:

- Solving equations, including linear systems with multiple unknowns
- Derivatives and rules of differentiation, for different types of functions
- Optimization and optimization under constraints (substitution method and Lagrange method)
- Comparative statics
- Concavity and convexity
- Game theory (basics)
    - Pure strategies, not mixed strategies
    - Dominant strategies and dominated strategies
    - Nash equilibrium
    - Backward induction and subgame perfection
    - Games of complete information (both discrete and continuous), not games of incomplete information.

Math II trains the following skills

- Graphical approach to linear programming
- Integrals and integration
- Difference equations
- Infinite sums
- Matrix algebra (basics: addition, multiplication, transposed, inverse, and symmetric matrix, determinant, Gaussian elimination, vector operations incl. inner product)

In the course Statistics I, no further math skills are taught, the courses uses some of the basic math skills listed above, mainly solving systems of equations.

Use of M&GT in other courses in Bachelor-1 and Bachelor-2

Using a survey, we asked the main lecturer of all other courses in bachelor-1 and bachelor-2 which skills were used in their course; the full survey can be found in the Appendix. In particular, for a number of skills, we asked:

- Which skills they expect students to know before they enter the course
- Which skills they explicitly teach and examine,
- Which skills they would like to see improved before students enter the course

We grouped the skills in these categories; the exact skills can be found in the Appendix:[1]

- basic algebra (3 skills)
- Derivatives, differentiation & optimization (8)[2]
- Optimization under constraints (4)
- Integrals & integration (3)[3]
- Limits (1)
- Infinite sums (1)
- Matrix algebra (4)
- Game theory (13).

The figures below depict for each of these categories to which extent these are used in a specific course. The courses are horizontally listed in order of appearance in the programme.

The score on the vertical axis gives, per category, the use of skills from this category as a fraction of the skills taught in Math or Microeconomics. For instance, when a course uses all skills related to differentiation apart from second-order optimization, its score on differentiation would be 7/8 = 0.875.

Note that we miss information for Fiscale Economie (block 1 in bachelor-2 for Dutch programme) and Toegepaste Statistiek / Applied Statistics I (block 2 of bachelor-2). Based on the PC students' perception, the study-guide and some previous exams, we conclude that in both courses math is used to a very limited extent and game theory is not used. Hence, we treat these courses as if no M&GT skills are used.

Figure 1 depicts the use of the math skills taught in Math I and Microeconomics in the other courses in bachelor-1 and 2.[4] It is clear from Figure 1 that algebra, differentiation, and –to a somewhat lower degree— constrained optimization are applied in various courses. This implies that students have the opportunity to practice these skills throughout the first two years of the programme.

[1] The survey also contained a question on differential equations. However, this technique is neither taught nor used in any course in bachelor-1 or bachelor-2. We did not explicitly ask for difference equations. None of the surveyed lecturers mentioned this as an omission.

[2] The survey contains nine questions in this category. However, sine and cosine functions are neither taught nor used in any course. We leave out this question.

[3] We leave out the question on integration by parts, as this is not used in any course.

[4] For ease of exposition, we group category limits under skills taught in Math II.

Figure 1: Use of algebra, differentiation, and constrained optimization

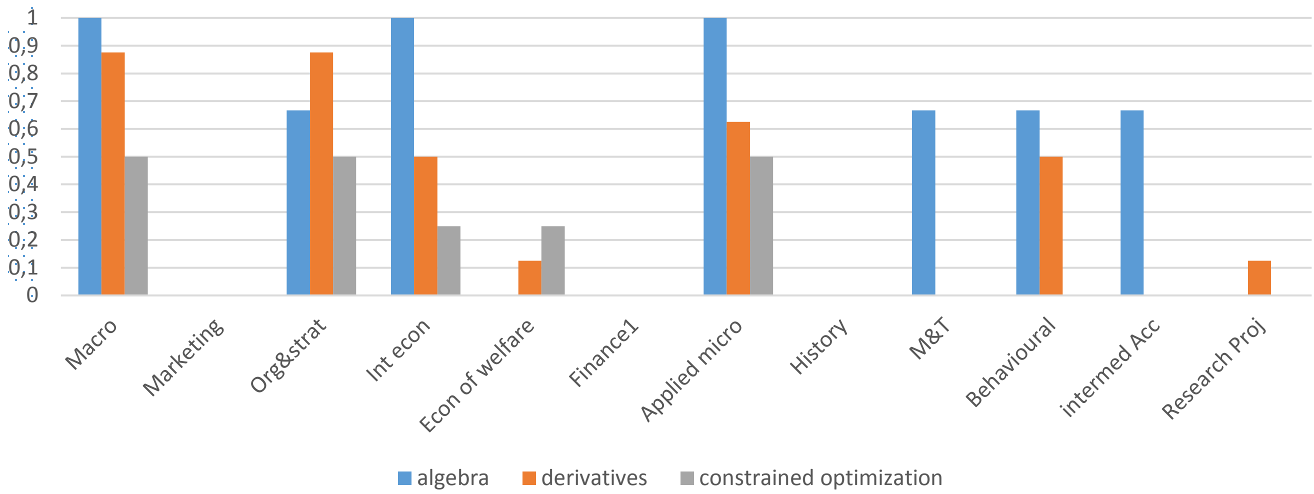

Figure 2: Use of Integration, Limits, infinite sums, and matrix algebra

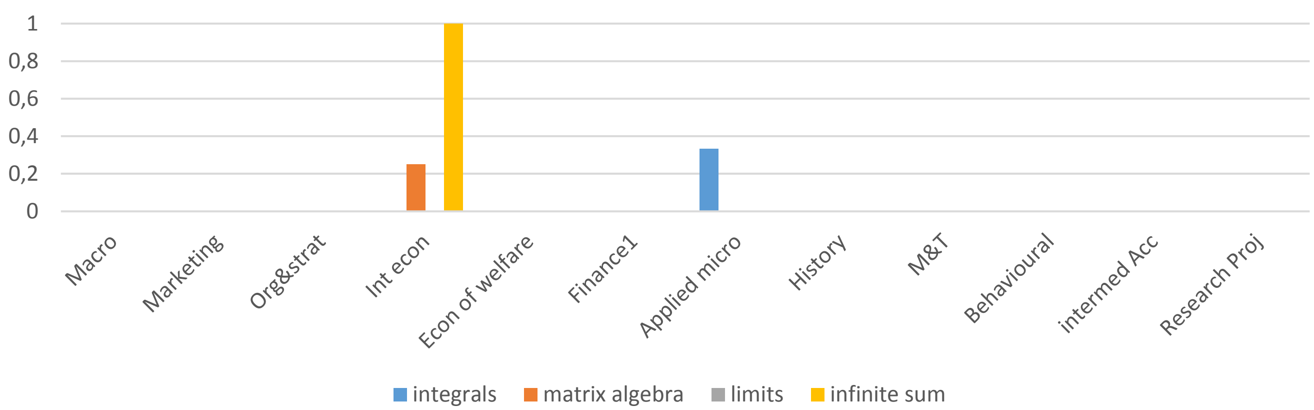

Figure 3: Use of game theory

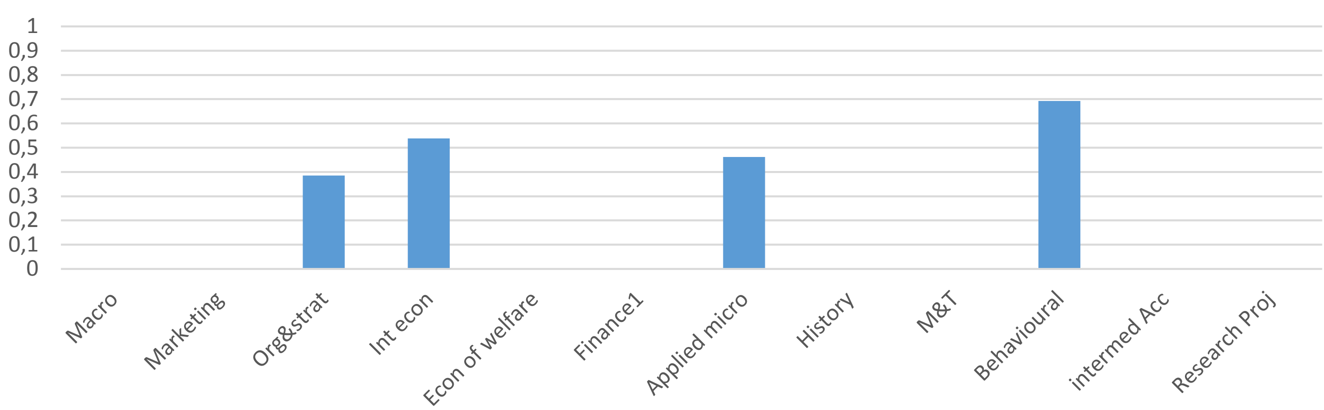

Figure 2 depicts the use of math skills taught in Math II. It follows that these skills are hardly used. Integrals are used only in graphical sense in one course, and never in mathematical sense. Matrix algebra and limits are never used. Infinite sums are used only in International Economics. Hence, students receive little to no opportunity to practice these skills before entering bachelor-3.

This poses a problem, also identified by the students as will be discussed in the focus groups below. Teaching these skills in Math II is only useful when students have the opportunity to practice the skills at various moments before their major in bachelor-3. If students do not get this opportunity, it would be better that these skills are taught at a later moment in the programme. This can be in the second part of bachelor-2, or in (a subset of) the majors.

In the survey, we asked the lectures of courses in bachelor-1 and 2 whether particular skills could be introduced in their course. This shows that in the current programme, only few lecturers see room to introduce some of these topics in their course:

- Integrals could be added to Applied Microeconomics
- Infinite sums could be used more extensively in Behavourial Economics.
- Apart from International Economics, matrix algebra cannot be introduced in any other course.
- Limits cannot be introduced in any course.

This is (clearly) not enough. Hence, we strongly recommend that there is coordination regarding the use of integration, limits, infinite sums, and matrix algebra in bachelor-1 and bachelor-2 courses. The outcome should be that students have enough opportunity to practice the use of skills taught in Math II. If for some skills, this cannot be organized, these skills should not be taught in block 3 of year 1 (where Math II takes place).

Figure 3 depicts the use of game theory in bachelor-1 and 2. Here, the score should be interpreted differently. Microeconomics teaches only the basics of game theory, and we have also asked whether more advanced concepts / skills were used in other courses (particularly regarding dynamic games and games with incomplete information). Hence, a score close to 0.7 reflects what is taught in Microeconomics, a higher score reflects that also more advanced material is taught.

Figure 3 shows that basic concepts in game theory are used in various courses throughout the bachelor. Hence, students can be expected to understand these concepts. Here, however, all these courses have game theory as a small part of the material, and it never reaches beyond the basics.

Conclusion and recommendations

Regarding math as taught in Math I and Microeconomics, this shows that the programme is coherent: the skills that are taught (mainly algebra, differentiation, and (constrained) optimization) are applied in various courses, balanced throughout bachelor-1 and 2. Hence, we recommend:

- Do not change the contents of Math I.

Regarding math as taught in Math II, this shows a mismatch: several skills (mainly integrals, matrix algebra, and limits) are taught, but not used for (at least) 1.5 years. Hence, changes are necessary here. We recommend the following:

- Coordinate an increase in the use of the math skills taught in Math II (integrals, matrix algebra, limits) in bachelor-2.

- Skills currently taught in Math II that cannot be integrated into bachelor-2 courses, should not be taught in Math II. Instead, the teaching of these skills should move either to the second part of bachelor-2 or to (a subset of) the majors.

Regarding game theory as taught in Microeconomics, this shows a fairly coherent pattern. Basic skills are used in various courses throughout the bachelor. Here, however, all these courses have game theory as a small part of the material, and it never reaches beyond the basics. Hence, we recommend the following:

- Coordinate how game theory is taught, and use the same approach, supported by webcasts.
- Consider dedicating a part of a course in bachelor-1 (or early bachelor-2) to game theory. Here, the general framework should be taught, which is then applied in various courses.

Lastly, we consider the information regarding which skills are used in which courses relevant for all lecturers. Hence, we recommend:

- Make this report available to all lecturers and students.
  Lecturers and students earlier in the programme can see when particular skills will be used in later courses. Lecturers can refer to this during the course. This increases relevance for students. Lecturers later in the programme can see which skills students are supposed to possess, and which skills need more attention.
- Give each lecturer reading access to the learning environment (blackboard / canvas) of all courses, at least of the courses in bachelor-1 and bachelor-2.

Use of math and game theory in majors (blocks 3 and 4 in bachelor-3)

Similar to the courses in bachelor-1 and bachelor-2, we used a survey to ask the main lecturer of all courses taught in blocks 3 and 4 bachelor-3. This covers all main courses for all majors. We did not survey the courses in block 2 of bachelor-3, as outgoing exchange students do not participate in those. We asked the same questions for the same skills as for the courses in bachelor-1 and bachelor-2; the full survey can be found in the Appendix. In particular, we asked for a list of skills:

- Which skills they expect students to know before they enter the course
- Which skills they explicitly teach and examine,
- Which skills they would like to see improved before students enter the course

We grouped the skills in these categories; the exact skills can be found in the Appendix:

- Basic algebra (3 skills)
- Derivatives, differentiation & optimization (8)[5]
- Optimization under constraints (3)
- Integrals & integration (4)
- Limits (1)
- Infinite sums (1)
- Differential equations (1)
- Matrix algebra (4)
- Game theory (13).

Response rates by major were generally good, as listed below (percentage is determined by the number of responses divided by the total number of major courses in block 3 and 4):

- Accounting and Finance (78%)
- Management Accounting (80%)
- Financial Accounting (60%)
- Financial Economics (100%)
- Economics of management and organization (100%)
- Policy economics (80%)
- International economics (100%)
- Marketing (50%)
- Industrial dynamics and strategy (80%)
- Urban, port, and transport economics (56%)

The outcomes of the survey is reported below in a number of figures. To keep these figures informative, we have decided to combine majors Management Accounting and Financial Accounting (denoted as ' Accounting'), as the overlap in courses is very large. For the same reason, we have decided not to report the major Accounting and Finance, as it consists of all courses in Financial and Management Accounting plus a subset of courses in Financial Economics.

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[5] The survey contains nine questions in this category. However, sine and cosine functions are neither taught nor used in any course. We leave out this question.

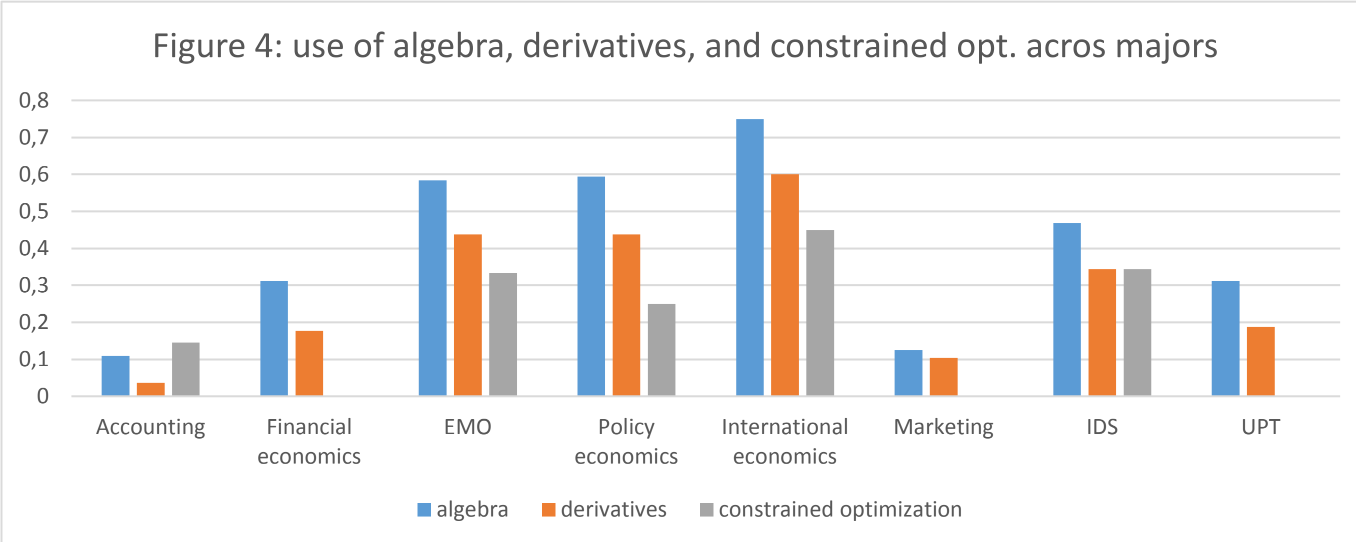
Figure 4: use of algebra, derivatives, and constrained opt. acros majors
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
Accounting
Financial economics
EMO
Policy economics
International economics
Marketing
IDS
UPT
algebra
derivatives
constrained optimization

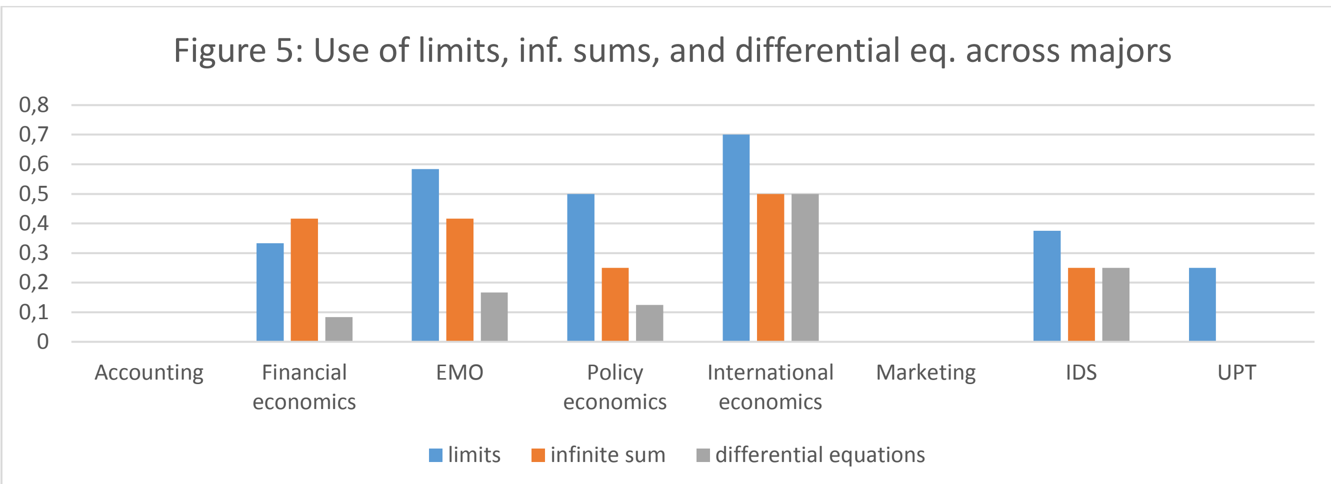
Figure 5: Use of limits, inf. sums, and differential eq. across majors
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
Accounting
Financial economics
EMO
Policy economics
International economics
Marketing
IDS
UPT
limits
infinite sum
differential equations

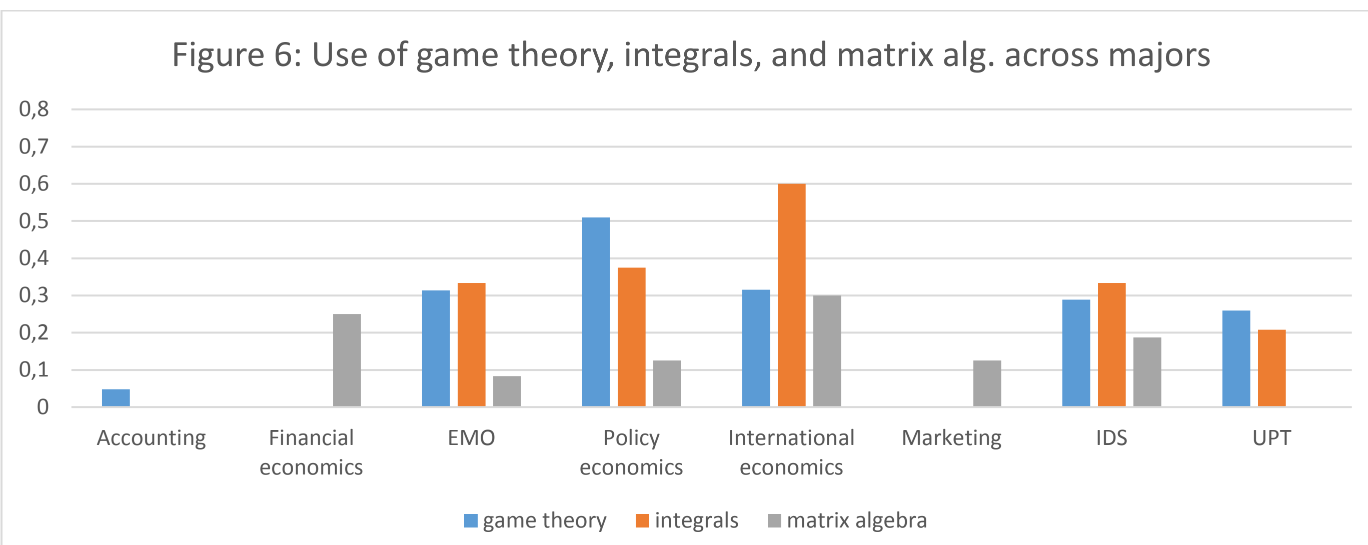
Figure 6: Use of game theory, integrals, and matrix alg. across majors
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
Accounting
Financial economics
EMO
Policy economics
International economics
Marketing
IDS
UPT
game theory
integrals
matrix algebra

Figures 4-6 depict for each of the skill categories to which extent these are used, separated by major. Here, used means either 'expect to be known' or ' taught and examined'. The score on the vertical axis gives, per category, the use of skills from this category as a fraction of the skills taught in Math or Microeconomics, averaged across the courses in the major.

In interpreting these figures, the exact fractions are not too interesting. Quite many major courses are predominantly empirical in nature. Majors with more empirical courses will have lower fractions. Hence, it is mainly interesting to see which skills are used and/or taught in many majors, and which skills in only a few.

Figure 4 shows the results for the use of algebra, derivatives and constrained optimization. Figure 5 shows the results for limits, infinite sums, and differential equations. Figure 6 shows the results for game theory, integrals, and matrix algebra. Again, these are used across many majors.

Figures 4 – 6 and inspection of the underlying data shows:

- Algebra and differentiation is used in all majors, constrained optimization is most.
- Limits, infinite sums, and differential equations are used in 5 different majors, although differential equations only to relatively limited extent. Note that differential equations are currently not taught in bachelor-1 or 2.
- Game theory, integrals, and matrix algebra are used across a subset of majors.

Preference for more training in skills

We also asked the lecturers which math and game theory skills they would like to see improved before students enter their course. The figures below report the outcomes, per skill category, by major. Here, the score on the vertical axis gives, per category, the fraction of skills lecturers would like to see improved, averaged across the courses in the major. Again, as many major courses are empirical in nature, the exact fractions are not too interesting.

Figure 7 shows the outcomes for algebra, derivatives and constrained optimization, Figure 8 shows the outcomes for limits, infinite sums, and differential equations, and Figure 9 shows the outcomes for game theory, integrals, and matrix algebra.

Figures 7 - 9 and inspection of the underlying data shows:

- Not surprisingly, across majors, preferences for improved skills are largely in line with the skills used and taught in the major.
- There is some concern among lecturers in general economic majors (EMO, policy, international economics) and in finance about algebraic skills and differentiation.
- Other than that, there is no clear pattern of subsets of majors demanding and increase in specific skills. There is some demand for better skills in nearly all categories.

Conclusion

Combining the outcomes in Figures 4 – 6 and Figures 7 – 9, we do not find skills that are taught in bachelor-1 and bachelor-2 that used only in one or two majors. Most M&GT skills are used widely. This holds for both the skills that are used in bachelor-2 as well as for the skills that are hardly used in bachelor-2.

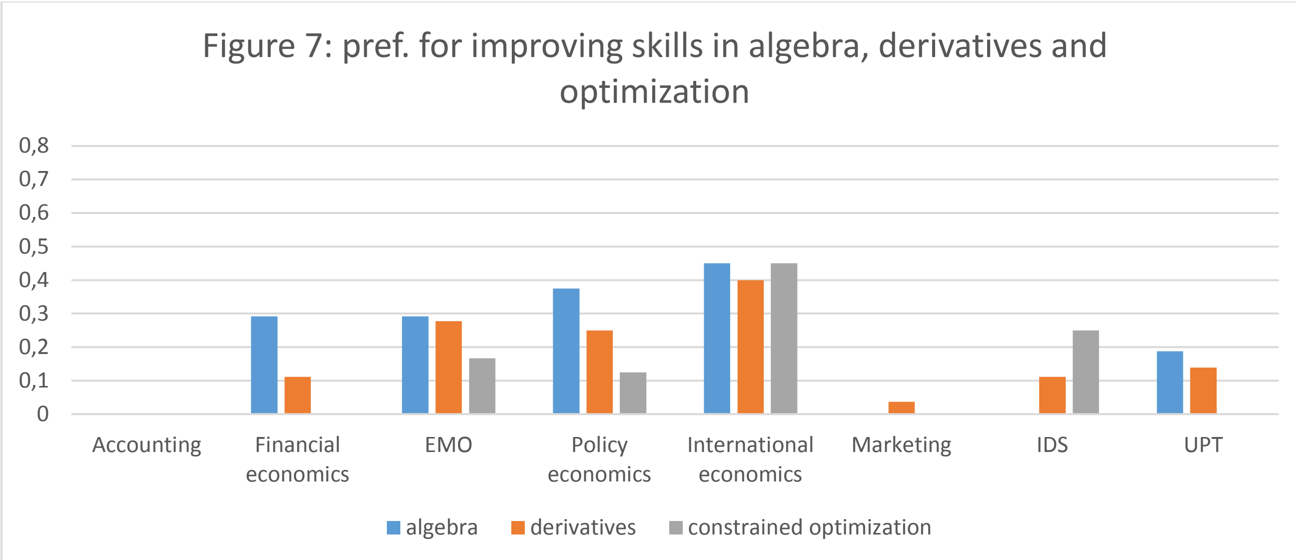
Figure 7: pref. for improving skills in algebra, derivatives and optimization
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
Accounting
Financial economics
EMO
Policy economics
International economics
Marketing
IDS
UPT
algebra
derivatives
constrained optimization

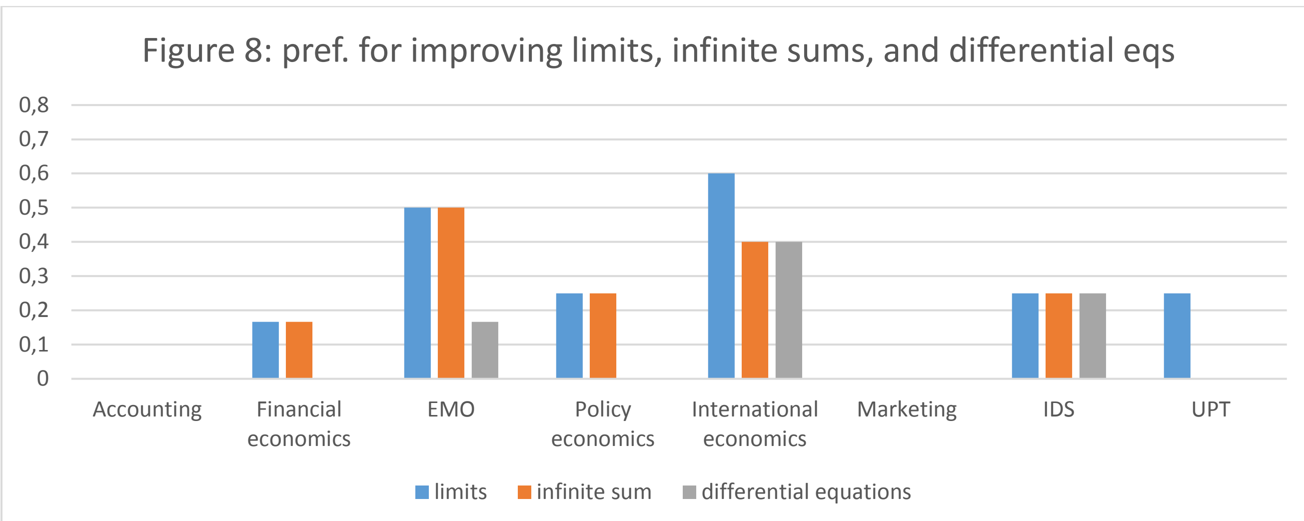
Figure 8: pref. for improving limits, infinite sums, and differential eqs
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
Accounting
Financial economics
EMO
Policy economics
International economics
Marketing
IDS
UPT
limits
infinite sum
differential equations

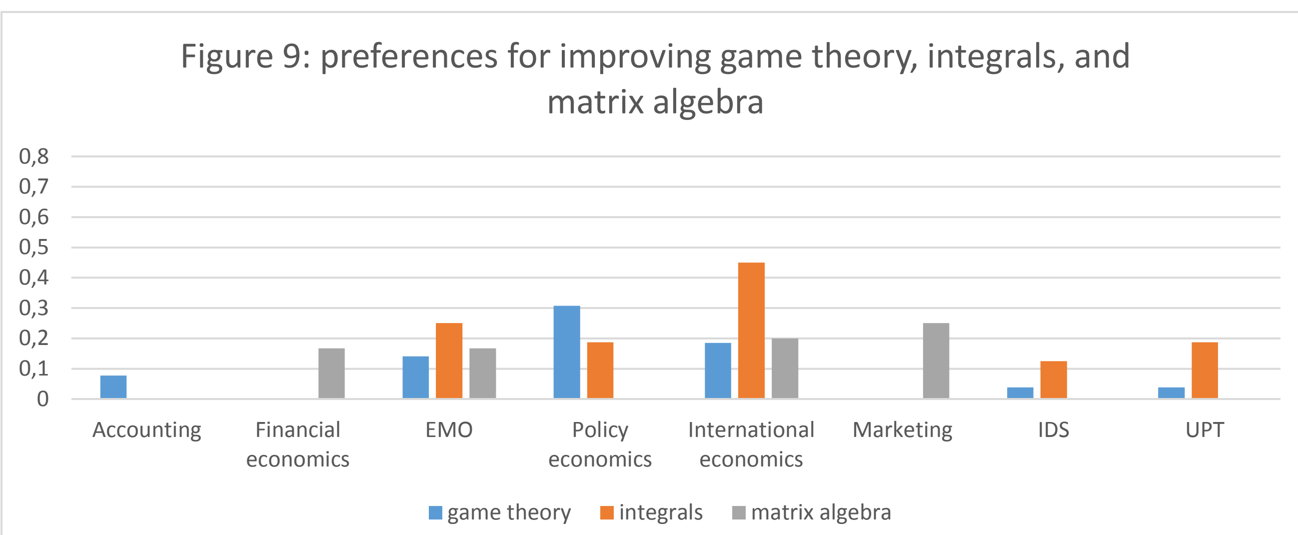
Figure 9: preferences for improving game theory, integrals, and matrix algebra
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
Accounting
Financial economics
EMO
Policy economics
International economics
Marketing
IDS
UPT
game theory
integrals
matrix algebra

Generally speaking, lecturers in the various major program s would prefer to see those skills improved that are used in their courses. There is not a clear demand for improved training on one specific skills (nor a general level of satisfaction with some skills).

As these M&GT skills are used widely, it is natural to train and use these skills in bachelor-1 and / or bachelor-2. While it might be possible to move some of these skills to major-specific courses, this seems not efficient. However, as documented above, some skills are currently taught in bachelor-1, but not used in bachelor-2, which is not effective. As will be discussed below, students also complain about this pattern.

Hence, this underlines the recommendations of the previous sections:

- Coordinate an increase in the use of the math skills taught in Math II (integrals, matrix algebra, limits) in bachelor-2.
- Skills currently taught in Math II that cannot be integrated into bachelor-2 courses, should not be taught in Math II. Instead, the teaching of these skills should move either to the second part of bachelor-2 or to (a subset of) the majors.

Subjective assessment of M&GT skills

We asked all lecturers of courses in a major to state to what extent they agreed with the following questions:

1. On average, the students enter my course with a sufficient level of math skills
2. On average, the students enter my course with a sufficient level of game theory skills
3. Among the students in my course, there is a lot of heterogeneity in the level of math skills
4. Among the students in my course, there is a lot of heterogeneity in the level of game theory skills
5. Our students have a sufficient level of math skills at the end of the bachelor programme Economics and Business Economics
6. Our students have a sufficient level game theory skills at the end of the bachelor programme Economics and Business Economics

Lectures could answer this on a 5-point likert scale, ranging from `very much disagree' to `very much agree'. 27 lecturers responded to these questions (although some did not answer the questions regarding game theory). The table below summarises the answers.

	mean	st.dev.	obs.
Course entry level math sufficient	3,23	0,71	27
Course entry level GT sufficient	2,95	0,58	22
Heterogeneity math level	4,08	0,89	25
Heterogeneity GT level	3,81	0,92	22
Final math level sufficient	3,10	0,83	22
Final GT level sufficient	3,19	1,59	17

This shows that most lecturers are quite neutral towards the entry level of math and game theory skills of their students. Remarkably, the heterogeneity of both math and game theory skills is considered to be high. Lectures agree to a limited degree with the statement that students possess sufficient math and game theory skills. As we will see, below, students express a similar view regarding the level of math taught and the heterogeneity in math skills among students at the ESE.

Conclusion

Overall, lecturers are neutral (but not very positive) about the level of M&GT. Lecturers do feel that the heterogeneity in math and game theory skills among students is large.

Focus groups among students in Bachelor-3

In order to hear directly from the students of the ESE, students of the Programme Committee organized focus groups during seminars in bachelor 3. The goal of these group sessions was to come into contact with students in bachelor 3 with different major specializations, and obtain their perspectives on math and game theory. During the focus group, we could talk with the students on a low key level about the use of math and game theory during their bachelor and in this way determine if there is an issue and where it arises.

At the start of block 4, we contacted the lecturers of all seminars, asking whether they could ask for a group of students to stay after a seminar session for a discussion on math. Sessions were held with students from 8 (out of 14) different seminars:

1. Financial Accounting and Reporting
2. Industrial Dynamics and Strategy
3. Corporate Finance
4. Stock Pricing and Corporate Events
5. Urban Port and Transport Economics
6. Competition Policy
7. Economics of the Public Sector
8. Interest Rates and Stock Markets

In the focus group sessions, we engaged in a structured discussion with about 4 to 7 students per session. The structure as prepared before conducting the sessions can be found in Appendix B. At the beginning of each session, we introduced the Programme Committee and the related math and game theory project. Following the session, we introduced the common themes. Central in this discussion was the experience of students on the use of math and game theory. Themes we were looking for were the level of math, a potential mismatch between what is taught and what is used and repeated discussion of the same techniques and concepts in different courses.
Below, we discuss the main findings, the feedback per session can be found in the appendix C.

Main Findings

We found that regarding the level of math taught, students had different opinions. Some students said that “the level of math is adequate” while others claimed that “the level is a bit too low” or even “really simple, very mechanical”. Multiple students in several groups expressed a desire for to further math knowledge, but noticed that the possibilities to do so are limited. One student remarked: “Changing Math I for all students could be too shocking and cause too many students to struggle, however in the current system extending math knowledge is not possible.” Another student suggested to ”give people the option to choose a more difficult track.”
Hence, the two main findings are that:

- Students’ perception of the level of math varies
- Some students would like the option to do more advanced math courses

On the potential mismatch between what is taught and what is used, a recurring theme was bachelor-2: “[The] problem of math is the second year. In the first year the math skills are fine, but there is only

Applied Microeconomics in the second year and some derivatives. Then in the third year the math is back." Students complained of a "gap" between learning mathematics early on in the bachelor and then seeing it come back later: "Skills disappear if these are not used steadily throughout the years."
In response, they suggested more use of mathematics in courses where this is applicable so that students have to keep using their skills. Other suggestions that were made to address this issue specifically were providing additional material, for example through webcasts.

In a few groups, student remarked that some parts of math taught is not used later: "you learn it [math] in bachelor-1, afterwards you never use it and forget it."

Hence, students feel that:

- There is too little use of math skills in bachelor-2, some skills (matrix algebra, integrals) are not used at all.
- More repetition would be helpful for retaining skills.
- (More) webcasts on math skills that students can consult in later courses would be helpful.

Regarding repeated discussion, a point that came up in nearly all focus groups is that math is "very mechanical. Students learn a trick without actually knowing what they're doing." To address this, students felt that more focus on intuition would help: this would form a stronger basis for further learning. In the words of one student: "Focus on intuition and understanding of math in the first year: why am I doing this, why does this work the way it does?" Furthermore, students would have liked that lecturers "when teaching math, explain the relevance more. Would be easier to apply what we learn if we understood it better." Because, " if you wait till the next year to start relating application to the theory, you might be too late."

Hence, student feel that:

- In teaching math skills to students, explain why they need to learn these skills:
    - Mention in which courses the skills will be used, and why.
    - When teaching / applying skills, also focus on the logic behind it.

Additional theme
In the focus group discussion, students used to opportunity to also raise other concerns, not directly related to math. This suggests that these focus groups can be used more broadly to actively find improvements for the education at the ESE.

One recurring theme raised during the sessions was the use of statistics and econometrics during the bachelor. In 5 out of the 8 seminar groups it was mentioned in the focus group. Perhaps this is due to the fact that math are statistics are related (both are seen as a 'skill' and are taught and used throughout the bachelor). Also, as the use of math and game theory differs by major, some groups naturally focussed more on the use of statistics. Even though this is not the main focus of the current report, we decide to report the main concerns, in the hope that this allows for better coordination.

Students' concerns regarding the courses on statistics and empirical methodology (mainly Methods & Techniques) revolved around two issues. First, students argue that there was " a mismatch between statistics and M&T" and that in "Applied Statistics we learned a lot of things that we never used

afterwards". Furthermore, while in one group students stated that "M&T was good", other groups complained that they were "just learning a trick", that you "can get away with limited knowledge", and that it "is quite superficial". In other words, they do not feel they are being prepared well. On top of that it is mentioned a couple of times that the statistics in this course are too theoretical.
These findings mirror those for math and game theory. The same goes for the recommendations of students: add explanations on the logic behind certain methods and making it more applied. So we can distinguish two subthemes that could be used in a follow-up of the current exercise:

- Logic behind statistics. How could this be achieved, would it help to link statistical methods more to the math behind it? This could also show the usefulness of matrix algebra, which is now felt as redundant (see above).
- Application of statistics: How can statistical models be linked to economics in general? Is this possible to do in a more general way, or is this too specific per major?

Conclusion and recommendations
The feedback from the focus groups was generally in line with the initial concerns. The perception of the level of math varied. Not many students struggled with math during their major (seminar), though that might be due to self-selection.
Nevertheless, the students pointed at several areas of improvement regarding math and game theory. Recommendations most often made were:

1. Several parts of math taught in bachelor-1 are used only in bachelor-3, some parts do not seem to be used.
   More training / repetition would be good, in particular in bachelor-2.
2. In the math courses, it is sometimes unclear why / when a particular technique is useful, students learn tricks rather than really understand what they are doing and why.
3. More online resources (e.g. webcasts that remain available) would be helpful. Note that creating webcasts can also be a response to point 2. All lecturers could contribute here, not only our Math and Microeconomics lecturers.
4. Some students would like an optional more difficult math course, for instance to prepare for seminars.

Lastly, students also pointed at points for improvement regarding statistics and empirical modelling. Here, the two main suggestions provided by students were:

5. Focus more on explaining the logic behind empirical models and statistics
6. Provide a better link between the theory of empirical models and their application in (various fields in) economics and business economics.

Hence, a follow-up of the current exercise focussed on statistics and empirical modelling would be valuable.

Concluding remarks

This reports documents the current use of math and game theory in the bachelor programme Economics and Business Economics. In the process of gathering information, we received many positive responses to our efforts from both lecturers and students. Many felt this was a useful exercise, and were eager to learn about the outcomes. Some expressed the hope that a similar exercise would be conducted for other parts of the curriculum.

As Programme Committee Economics and Business, we would like to thank all those who have contributed their time to this project.

Appendix A

Survey on the use of math and game theory in bachelor Economics and Business Economics

Following a discussion initiated by the students in the Education Committee Economics, the EC has decided to map the current use of math and game theory in the bachelor economics and business economics. This survey aims to find out which math and game theory skills are currently used in courses in block 3 and 4 of Bachelor-3.

Considering your course in bachelor-3, we would like to know (i) which math and game theory skills students are supposed to have before they enter your course, (ii) which math and game theory skills you teach (and examine) in your course, and (iii) which math and game theory skills you would like to see improved (before students enter your course).

Please note that in this project, we focus on math and game theory, ignoring e.g. statistics and empirical skills.

Below is a (non-exhaustive) list of math and game theory skills. Please check the relevant box. At the end, there is space to add skills you teach or would like to students to possess that are not in the list. Please return the form to Josse Delfgaauw, H9-23, delfgaauw@ese.eur.nl

Please enter the course code and name

Skills	(i) Expect students to have before entering the course	(ii) Taught and examined in the course	(iii) Would like to see improved before entering the course
Algebra			
- Solving equations	☐ yes ☐ no	☐ yes ☐ no	☐ yes ☐ no
- Implicit functions	☐ yes ☐ no	☐ yes ☒ no	☐ yes ☐ no
- Concavity / convexity	☐ yes ☐ no	☐ yes ☐ no	☐ yes ☐ no
Differentiation / derivatives			
- Power functions	☐ yes ☐ no	☐ yes ☐ no	☐ yes ☐ no
- Log / exponential	☐ yes ☐ no	☐ yes ☐ no	☐ yes ☐ no
- Chain rule	☐ yes ☐ no	☐ yes ☐ no	☐ yes ☐ no
- Sin / cosine etc.	☐ yes ☐ no	☐ yes ☐ no	☐ yes ☐ no
- Inverse function differentiation	☐ yes ☐ no	☐ yes ☐ no	☐ yes ☐ no
- Implicit differentiation	☐ yes ☐ no	☐ yes ☐ no	☐ yes ☐ no
- Comparative statics	☐ yes ☐ no	☐ yes ☐ no	☐ yes ☐ no

- First-order condition	☐ yes ☐ no	☐ yes ☐ no	☐ yes ☐ no
optimization	☐ yes ☐ no	☐ yes ☐ no	☐ yes ☐ no
- Second-order condition optimization	☐ yes ☐ no	☐ yes ☐ no	☐ yes ☐ no
Optimization under constraints			
- Graphical method	☐ yes ☐ no	☐ yes ☐ no	☐ yes ☐ no
- Substitution method	☐ yes ☐ no	☐ yes ☐ no	☐ yes ☐ no
- Lagrangian method	☐ yes ☐ no	☐ yes ☐ no	☐ yes ☐ no
- Kuhn-Tucker method	☐ yes ☐ no	☐ yes ☐ no	☐ yes ☐ no
Integrals / integration			
- Graphical analysis / area's under functions	☐ yes ☐ no	☐ yes ☐ no	☐ yes ☐ no
- Integration by substitution	☐ yes ☐ no	☐ yes ☐ no	☐ yes ☐ no
- Integration over infinite interval	☐ yes ☐ no	☐ yes ☐ no	☐ yes ☐ no
- Integration by parts	☐ yes ☐ no	☐ yes ☐ no	☐ yes ☐ no
Limits	☐ yes ☐ no	☐ yes ☐ no	☐ yes ☐ no
Infinite sums	☐ yes ☐ no	☐ yes ☐ no	☐ yes ☐ no
Differential equations	☐ yes ☐ no	☐ yes ☐ no	☐ yes ☐ no
Matrix algebra			
- Adding / multiplication	☐ yes ☐ no	☐ yes ☐ no	☐ yes ☐ no
- Transpose	☐ yes ☐ no	☐ yes ☐ no	☐ yes ☐ no
- Inverse matrix	☐ yes ☐ no	☐ yes ☐ no	☐ yes ☐ no
- Inner product	☐ yes ☐ no	☐ yes ☐ no	☐ yes ☐ no
Game theory			
- Pure strategies	☐ yes ☐ no	☐ yes ☐ no	☐ yes ☐ no
- Nash equilibrium	☐ yes ☐ no	☐ yes ☐ no	☐ yes ☐ no
- Dominant strategies	☐ yes ☐ no	☐ yes ☐ no	☐ yes ☐ no
- Iterative elimination of dominated strategies	☐ yes ☐ no	☐ yes ☐ no	☐ yes ☐ no

- Maxi-min strategy	☐ yes ☐ no	☐ yes ☐ no	☐ yes ☐ no
- Mixed strategies	☐ yes ☐ no	☐ yes ☐ no	☐ yes ☐ no
- Games of complete information, discrete (e.g. prisoners dilemma)	☐ yes ☐ no	☐ yes ☐ no	☐ yes ☐ no
- Games of complete information, continuous (e.g. cournot)	☐ yes ☐ no	☐ yes ☐ no	☐ yes ☐ no
- Game with multiple equilibria (e.g. coordination games)	☐ yes ☐ no	☐ yes ☐ no	☐ yes ☐ no
- Games with incomplete information	☐ yes ☐ no	☐ yes ☐ no	☐ yes ☐ no
- Dynamic games	☐ yes ☐ no	☐ yes ☐ no	☐ yes ☐ no
- Backward induction	☐ yes ☐ no	☐ yes ☐ no	☐ yes ☐ no
- Subgame perfect equilibrium	☐ yes ☐ no	☐ yes ☐ no	☐ yes ☐ no

Please give your opinion on the following statements:

On average, the students enter my course with a sufficient level of math skills	☐ Very much agree ☐ Agree ☐ neutral ☐ Disagree ☐ Very much disagree
On average, the students enter my course with a sufficient level of game theory skills	☐ Very much agree ☐ Agree ☐ neutral ☐ Disagree ☐ Very much disagree
Among the students in my course, there is a lot of heterogeneity in the level of math skills	☐ Very much agree ☐ Agree ☐ neutral ☐ Disagree ☐ Very much disagree ☐No opinion / do not know
Among the students in my course, there is a lot of heterogeneity in the level of game theory skills	☐ Very much agree ☐ Agree ☐ neutral ☐ Disagree ☐ Very much disagree
Our students have a sufficient level of math skills at the end of the bachelor programme Economics and Business Economics	☐ Very much agree ☐ Agree ☐ neutral ☐ Disagree ☐ Very much disagree ☐No opinion / do not know

Our students have a sufficient level game theory skills at the end of the bachelor programme Economics and Business Economics	☐ Very much agree ☐ Agree ☐ neutral ☐ Disagree ☐ Very much disagree ☐No opinion / do not know

Please add any comments and suggestions regarding math and game theory skills:

Appendix B: focus group structure

Approximate duration: 30 minutes

The focus group will be structured around:

- Two subjects:
    1. Math
    2. Models
- Three dimensions:
    1. Current level of math/models
    2. Possible mismatch between what is taught and what is used
    3. Possible lack of coordination

The content of the focus group will be an introduction, questions (engagement questions, exploration questions and exit questions) and a conclusion.

Important remarks will be summarized for every question.

Introduction:
Introduce ourselves and the Education Committee. Introduce the project of the EC regarding math and models. Explain the design of the focus group and the subject of math and models.

Questions:
Engagement questions:

1. What prior math/models have you come across during your bachelor?
2. What is taught and when is it taught?
3. What was your experience?

Exploration questions:

4. How do you feel about the level of what is taught?
5. Is what is taught adequate for your major and Master choice?
6. Is what is taught complete? Does it cover all relevant aspects?
7. How is the coordination between courses that teach or require math and models skills?
8. Does what is taught correspond to what is needed for your major and Master choice?

Exit questions:

9. If you had any difficulties, where do these come from?
10. How would you solve these difficulties?

Conclusion:
Goal: Tell students what is done with the feedback.
The feedback is gathered for all possible seminars and evaluated in May for an advice to programme management regarding math and models in courses (both what is used and what is taught).

Possible relevant courses:

Math	Models
•Math I (B1) •Math II (B2) •*Applied Microeconomics (B2)*	•Microeconomics (B1) •Macroeconomics (B1) •*International Economics (B2)* •*Applied Microeconomics (B2)*

For information:

Skills
Algebra - Solving equations - Implicit functions - Concavity / convexity
Differentiation / derivatives - Power functions - Log / exponential - Chain rule - Sine / cosine etc. - Inverse function differentiation - Implicit differentiation - Comparative statics - First-order condition optimization - Second-order condition optimization
Optimization under constraints - Graphical method - Substitution method - Lagrangian method - Kuhn-Tucker method
Integration / integrals - Graphical analysis / areas under functions - Integration by substitution - Integration over infinite interval - Integration by parts
Limits
Infinite sums
Differential equations
Matrix algebra - Adding / multiplication - Transpose - Inverse matrix - Inner product
Game theory - Pure strategies - Nash equilibrium - Dominant strategies

- Iterative elimination of dominated strategies
- Maximin strategy
- Mixed strategies
- Games of complete information, discrete (e.g. prisoner’s dilemma)
- Games of complete information, continuous (e.g. Cournot)
- Game with multiple equilibria (e.g. coordination games)
- Games with incomplete information
- Dynamic games
- Backward induction
- Subgame perfect equilibrium

Appendix C: Feedback per session

1. Financial Accounting and Reporting

Statistics:

- Feedback: students are just learning a trick. Redo exercise without knowing the logic. The students mention the open book exam in statistics as an example, in which you would only have to know which pages are relevant.

General:

- Students need an explanation behind an approach, learn the why of certain approaches and the logic behind it.

2. Industrial Dynamics and Strategy

Math and models:

- Mostly an issue during the third year. In general, level is adequate. More of a mismatch between statistics and M&T.
- Matrices and integration are never used.
- Math I is more linear, the logic makes sense when reading papers now (useful).
- Summer course was really helpful (good for new students).

Statistics:

- You can get away with limited knowledge about models in the Research Project.
- M&T is too much theory, not applied enough, now using YouTube instead.

General:

- Positive about webcasts, maybe cases to solve?

3. Corporate Finance

Math and models:

- Level of math is really low, level in high school was higher. The math was really simple, very mechanical. Students learn a trick without actually knowing what they're doing.
- Some people seem to struggle with mathematics, especially people from the UK and NL.
- Give people the option to choose a more difficult track.
- Use of matrices for portfolio maximization only in seminar, not before.
- Pre-Calculus (econometrics) comparable to Math I and Math II. The courses Calculus and Matrix Algebra would be good additions to the economics curriculum?
- Not necessarily major-specific, would help overall. However 'easier math version' available.
- The problem is not that the math is too theoretical, it is very applied. Calculus [econometrics course] starts with the very basis, but for Math I and Math II the beginning and ending part is missing. They give you something, you are forced to believe it and don't know why. Give students more intuition (We know how to calculate a beta but not what it is).
- Use a logical order in math education and build from the basics.
- Standardize the mathematics taught, as they do in Delft (lot of studies have math courses). Expectation is a base knowledge on mathematics, but boost the level of mathematics overall.
- Something in between economics and econometrics would be great.
- An idea would be to have a certificate for doing several additional math courses.

- Map what knowledge is expected for different courses, such as Quantitative Finance. Get the baseline higher, and offer additional courses to those who are motivated.

<u>4. Stock Pricing and Corporate Events</u>
Math and models:

- Level is adequate. Very little math is used after Math I and Math II in the first year.
- It is a tough step to econometrics. When you start with econometrics courses you never get the introduction so you never get the basic knowledge which is necessary.
- Online course for those who want to raise their math level would be really nice.
- Does not need to be as in-depth as for econometrics, but understanding it would be helpful.
- General impression: you learn it in bachelor-1, afterwards you never use it and forget it.
- Use it more in other subjects; make other courses more math-oriented. Right now, there are a lot of courses where math is not used at all.

Statistics:

- M&T was quite superficial, only one exercise with loops but they never explained how to use them. M&T was too theoretical, there was not enough bringing it into practice.
- Applied Statistics we learned a lot of things that we never used afterwards, but regarding Stata or practical things we didn't learn very much.
- Use SPSS in the first year and then switch to Stata, why?

<u>5. Urban Port and Transport Economics</u>
Math and models:

- Level is a bit too low. In econometrics the level is very high and in economics it is relatively low. There is a large gap between the two, for which nothing is available. Therefore it is hard for motivated students to upgrade their math skills.
- Math I and Math II are not so hard for students coming from IB or for international students, though the Dutch Math A students find it tough, and often have to do a resit.
- An option to choose more difficult mathematics would be nice.
- Additionally, filtering the level of mathematics – mathematics backgrounds differ a lot – might help differentiate between students who enjoy math and want to go further and those that don't.
- Perception of people outside the ESE is that there is a lot of mathematics, but this is not really the case in practice.
- Two years without any mathematics (between bachelor 1 blocks and bachelor 3) is very long.
- Changing Math I for all students could be too shocking and cause too many students to struggle, however in the current system extending math knowledge is not possible.
- Papers from economics are often based on econometric models, but it is hard to understand these. Would not be necessary to have the skills to create the models, but at least to understand them. Simple concepts from econometrics would help.
- Second and third block in bachelor 3 are not really related to math – adding more math here could prepare for the major seminar and thesis.
- Would be a good idea to work on this on a major-specific level.
- Would like to be able to choose more math-oriented majors or masters instead of what is currently available.
- Inclusion of some econometrics would be nice, especially in the third year.

- Math I and Math II focus on theory, but if you wait till the next year to start relating application to the theory, you might be too late. Current setup focuses on learning models without application.
- Keep recap short, so that the things we know aren't taught over and over. This gives more time to go in-depth in the course.
- Students have different preferences regarding mathematics, some like it more and some like it less – differentiate and challenge students.
- One small course with some recap prior to the thesis on mathematics/models/methods towards the end of the third year would be nice.

Statistics:

- M&T was good, but there could be more statistics.
- Long time between thesis and M&T.
- Online recap? Only if the professor really knows how to use it. Refresh knowledge.
- In statistics, different programs are used in the different years: SPSS in bachelor 1, then Stata in bachelor 2.

<u>6. Competition Policy</u>

Math and models:

- Problem of math is the second year. In the first year the math skills are fine, but there is only Applied Microeconomics in the second year and some derivatives. Then in the third year the math is back.
- Economics of Markets and Organizations which requires a lot of math is not mandatory, and not necessary for all seminars so students can get by for the bachelor.
- Topics like matrices came back in finance, but what you use depends very much on the seminar you choose.
- Math level high enough though some feel it is insufficient.
- Repetition can be good for the first and second year math.
- Would like an extra course to boost math skills, perhaps credits or certificate?
- When teaching math, explain the relevance more. Would be easier to apply what we learn if we understood it better.
- Liked the webcasts in the math courses.
- If there is more intuition in the first year, then you keep it with you.
- Lectures are only two hours, but if students want to know intuition or application additional information through webcasts could be provided.

Comment Kamphorst: some students of this seminar would really be interested in courses/additional material in between economics and econometrics.

<u>7. Economics of the Public Sector</u>

Math and models:

- There is a long period between utilizing math/models in the seminar and actually learning it.
- We only get applied mathematics in some courses.
- Additional material would be a huge improvement, such as online tutorials or links to explain how to do certain math.
- Emphasize in the first year and second year courses that what they're teaching you is not just a trick, but that it will return.

- Combine this with a more ‘applied’ guide or webcast.
- Major specific issue? Choice for students together with pre-requisites for courses.

Statistics:

- You could in Methods and Techniques look at papers which use the techniques and discuss why they are used in these cases.
- Subject in between Methods and Techniques and seminar would help: there are some such as Impact Evaluation and Financial Methods and Techniques.
- In Methods and Techniques, add specific cases to work on. Now you don’t learn what it means. You get effective at looking things up, but not in learning.

General:

- Lot of courses have a little trick, so we don’t retain a lot.

<u>8. Interest Rates and Stock Markets</u>

Math and models:

- Math is relevant, everything is becoming more quantitative and data-driven.
- More practice and application would be nice.
- For this seminar we had two assignments where we could choose between Stata and Excel, but this was explained well. Also, we used some matrix algebra from Math II.
- Not a lot of people who had trouble with math in the seminar.
- In the seminar, not really desire for more math.
- How math is taught (as TA):The theory is explained once or twice, but we don’t focus on it and we’re not tested based on it; only have to apply knowledge. If you ask students, they often don’t know theory.
- Focus on intuition and understanding of math in the first year: why am I doing this, why does this work the way it does?
- Skills disappear if these are not used steadily throughout the years.
- Offer more opportunities to apply knowledge.
- Would additional webcasts help? Relatively easy to do this in Math I and Math II: offer an example of how to apply the knowledge in a real-world case at the end of every lecture.
- Offer more repetition possibilities for students who want this, or possibilities to apply theory to real-world examples.