Exercises on entry level (Instapniveau) for the course Introduction to Analysis

Exercise 1.

Solve the following inequalities and write the answer in one of the following forms: $a < x < b$, $a < x \leq b$, $a \leq x < b$, $a \leq x \leq b$.

a) $$\frac{3}{x-1} - \frac{4}{x} \geq 1$$

$$\frac{3x - 4(x-1)}{x(x-1)} \geq 1 \quad \rightarrow \quad \frac{4-x}{x(x-1)} \geq 1$$

$\rightarrow$ ① if $x \in (-\infty, 0) \cup (1, +\infty)$ then $4-x \geq x(x-1)$

$\rightarrow$ ② if $x \in (0,1)$ then $4-x \leq x(x-1)$

Case ①: $x \in (-\infty, 0) \cup (1, +\infty)$

$$4-x \geq x(x-1)$$
$$4-x \geq x^2 - x$$
$$4 \geq x^2$$
$$-2 \leq x \leq 2 \quad \text{or} \quad x \in [-2, 2]$$

Now $[-2,2] \cap \{(-\infty, 0) \cup (1, +\infty)\}$

$= [-2, 0) \cup (1, 2]$

Case ②, $x \in (0,1)$

$$4-x \leq x(x-1)$$
$$4-x \leq x^2 - x$$
$$4 \leq x^2$$

Hence $x \in (-\infty, -2] \cup [2, +\infty)$

But $\{(-\infty, -2] \cup [2, +\infty)\} \cap (0,1) = \emptyset$

The overall solution is $x \in \{[-2, 0) \cup (1, 2]\} \cup \emptyset$

$\Rightarrow x \in [-2, 0) \cup (1, 2]$

$\rightarrow$ $-2 \leq x < 0$ or $1 < x \leq 2$

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b) $8 - |2x-1| \geq 6$

$-|2x-1| \geq -2$

$|2x-1| \leq 2$

Case ① if $x \in [1/2, +\infty)$

$|2x-1| \leq 2$

$2x-1 \leq 2$

$2x \leq 3$

$x \leq 3/2 \Rightarrow x \in (-\infty, 3/2]$

$x \in [1/2, +\infty) \cap (-\infty, 3/2]$

$\Rightarrow x \in [1/2, 3/2]$

Case ② if $x \in (-\infty, \frac{1}{2})$

$|2x-1| \leq 2$

$-2x+1 \leq 2$

$-2x \leq 1$

$x \geq -\frac{1}{2} \Rightarrow x \in [-\frac{1}{2}, +\infty)$

$x \in (-\infty, 1/2) \cap [-1/2, +\infty)$

$\Rightarrow x \in [-1/2, 1/2)$

The overall solution is $x \in [1/2, 3/2] \cup [-1/2, 1/2)$

$\therefore x \in [-1/2, 3/2]$

$\therefore -\frac{1}{2} \leq x \leq 3/2$

c)

$$\left|\frac{x-1}{x+1}\right| \leq 1 \iff -1 \leq \frac{x-1}{x+1} \leq 1$$

Case ① if $x \in (-\infty, -1)$ then $x+1 < 0$

$$-1 \leq \frac{x-1}{x+1} \leq 1 \Rightarrow -1(x+1) \geq x-1 \geq x+1$$

$$\Rightarrow -x-1 \geq x-1 \geq x+1$$

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i) $-x-1 \geq x-1$

$0 \geq 2x$

$0 \geq x$

ii) $x-1 \geq x+1$

$0 \geq 2$

$\hookrightarrow$ Statement is false ✗

Case ② If $x \in [-1, +\infty)$ then $x+1 > 0$

$$-1 \leq \frac{x-1}{x+1} \leq 1 \Rightarrow -x-1 \leq x-1 \leq x+1$$

i) $-x-1 \leq x-1$

$0 \leq 2x$

$0 \leq x \Rightarrow x \in [0, +\infty)$

ii) $x-1 \leq x+1$

$0 \leq 2$

$\hookrightarrow$ Statement is true ✓

$\Rightarrow \quad x \in [0, +\infty) \cap [-1, +\infty)$

$\therefore \; x \in [0, +\infty)$

The overall solution is $x \in [0, +\infty)$

$\therefore \quad 0 \leq x$

Exercise 2.

Determine all possible values of x for which the following expressions will be real numbers:

a) $\left(\frac{1}{x^2-x-1}\right)^{\frac{1}{2}}$

① $x^2-x-1=0$

$\Rightarrow$ use of $\frac{-b \pm \sqrt{b^2-4ac}}{2a} = x_{1,2}$

$\Rightarrow \quad x_1 = \frac{1-\sqrt{5}}{2} \qquad x_2 = \frac{1+\sqrt{5}}{2}$

$$x^2-x-1 = \left(x - \frac{1-\sqrt{5}}{2}\right)\left(x - \frac{1+\sqrt{5}}{2}\right)$$

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$$f(x) = \left(\frac{1}{x^2-x-1}\right)^{1/2} = (g(x))^{1/2} \quad \text{therefore} \quad g(x) \geq 0$$

$$g(x) = \frac{1}{x^2-x-1} \geq 0 \iff g(x) \frac{1}{\left(x-\frac{1-\sqrt{5}}{2}\right)\left(x-\frac{1+\sqrt{5}}{2}\right)} \geq 0$$

Case 1 $x \in \left(-\infty, \frac{1-\sqrt{5}}{2}\right) \cup \left(\frac{1+\sqrt{5}}{2}, +\infty\right)$

$1 \geq 0$ The statement is TRUE ✓

Case 2 $x \in \left(\frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right)$

$1 \leq 0$ The statement is FALSE ✗

The overall solution is $x \in \left(-\infty, \frac{1-\sqrt{5}}{2}\right) \cup \left(\frac{1+\sqrt{5}}{2}, +\infty\right)$

b)

$$\sqrt{x - \sqrt{|x-1|}}$$

$$f(x) = \left(x - (|x-1|)^{1/2}\right)^{1/2}$$
$$= (g(x))^{1/2} \quad \text{where } g(x) = x - |x-1|^{1/2}$$

$\Rightarrow$ It must be that $g(x) \geq 0$, thus $x - |x-1|^{1/2} \geq 0$

$$x \geq |x-1|^{1/2} \iff x^2 \geq |x-1|$$

Case 1 if $x \in [1, +\infty)$

$x \geq (x-1)^{1/2}$

$x^2 - x + 1 \geq 0$

$\Delta = 1 - 4 = -3$ $\Big\} \Rightarrow$ $x^2 - x + 1 \geq 0$ $\forall x \in \mathbb{R}$

$\Rightarrow x \in [1, +\infty) \cap [1, +\infty)$

$\Rightarrow x \in [1, +\infty)$

Case 2 if $x \in (-\infty, 1)$

$x \geq (-x+1)^{1/2}$

$\therefore x^2 \geq -x + 1 \iff x^2 + x - 1 \geq 0$

$\left(x + \frac{-1+\sqrt{5}}{2}\right)\left(x + \frac{-1-\sqrt{5}}{2}\right) \geq 0$

$\therefore x \in \left[\frac{-1+\sqrt{5}}{2}, +\infty\right) \cap (-\infty, 1)$

$\therefore x \in \left[\frac{-1+\sqrt{5}}{2}, 1\right)$

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The overall solution is $x \in [1, +\infty) \cup \left[\frac{-1+\sqrt{5}}{2}, 1\right)$

$\therefore\ x \in \left[-\frac{1+\sqrt{5}}{2}, +\infty\right)$

c) $\ln(4x - |4x^2 - 1|)$

$f(x) = \ln(4x - |4x^2 - 1|) = \ln(g(x))$

where $g(x) = 4x - |4x^2 - 1|$, s.t. $g(x) > 0$

hence $4x - |4x^2 - 1| > 0 \Leftrightarrow 4x > |4x^2 - 1|$

Case 1 for $x \in \left(-\frac{1}{2}, \frac{1}{2}\right)$

$4x > -4x^2 + 1$

$4x^2 + 4x - 1 > 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = x_{1,2}$

$\frac{-4 \pm 4\sqrt{2}}{8} = x_{1,2}$

$\Rightarrow x \in \left\{\left(-\infty, \frac{-4-\sqrt{32}}{8}\right) \cup \left(\frac{-4+\sqrt{32}}{8}, +\infty\right)\right\} \cap \left(-\frac{1}{2}, \frac{1}{2}\right)$

$\Rightarrow x \in \left(\frac{-1+\sqrt{2}}{2}, \frac{1}{2}\right)$

Case 2 for $x \in \left(-\infty, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, +\infty\right)$

$4x > 4x^2 - 1$

$0 > 4x^2 - 4x - 1$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = x_{1,2}$

$\frac{4 \pm 4\sqrt{2}}{8} = x_{1,2}$

$\Rightarrow x \in \left(\frac{4-\sqrt{32}}{8}, \frac{4+\sqrt{32}}{8}\right) \cap \left(-\infty, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, +\infty\right)$

$\Rightarrow x \in \left[\frac{1}{2}, \frac{1+\sqrt{2}}{2}\right)$

The overall solution is $x \in \left(\frac{-4+\sqrt{32}}{8}, \frac{1}{2}\right) \cup \left[\frac{1}{2}, \frac{4+\sqrt{32}}{8}\right)$

$x \in \left(\frac{-1+\sqrt{2}}{2}, \frac{1+\sqrt{2}}{2}\right)$

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Exercise 3.

For which real numbers x does the inequality $f(x) \geq g(x)$ hold? Given

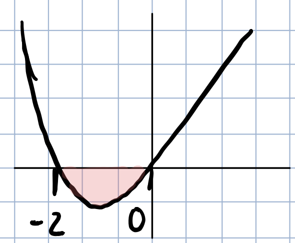
$$f(x) = x(x+2) \quad \text{and} \quad g(x) = 2x^2 + 4x.$$

$$f(x) \geq g(x) \iff x(x+2) \geq 2x^2 + 4x$$

$$x^2 + 2x \geq 2x^2 + 4x$$

$$0 \geq x^2 + 2x$$

$$0 \geq x(x+2)$$



$$\therefore \; x \in [-2, 0]$$

Exercise 4.

Show that the points *A(1, 1)*, *B(7, 4)*, *C(5, 10)* and *D(-1, 7)* in the plane are in fact the vertices of a parallelogram, by purely making use of the slopes of the connecting lines between the vertices.

If the shape ABCD is a parallelogram, then the line segments AB and CD must be parallel to each other! Furthermore the line segments DA and BC must be parallel! We know two lines are parallel if the slopes of the two line segments are equal!

① $$\text{Slope}_{AB} = m_{AB} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-1}{7-1} = \frac{3}{6} = \frac{1}{2}$$

$$\text{Slope}_{CD} = m_{CD} = \frac{7-10}{-1-5} = \frac{-3}{-6} = \frac{1}{2}$$

$$m_{AB} = m_{CD}$$

② $$\text{Slope}_{DA} = m_{DA} = \frac{1-7}{1+1} = \frac{-6}{2} = -3$$

$$\text{Slope}_{BC} = m_{BC} = \frac{10-4}{5-7} = \frac{6}{-2} = -3$$

$$\rightarrow m_{DA} = m_{BC}$$

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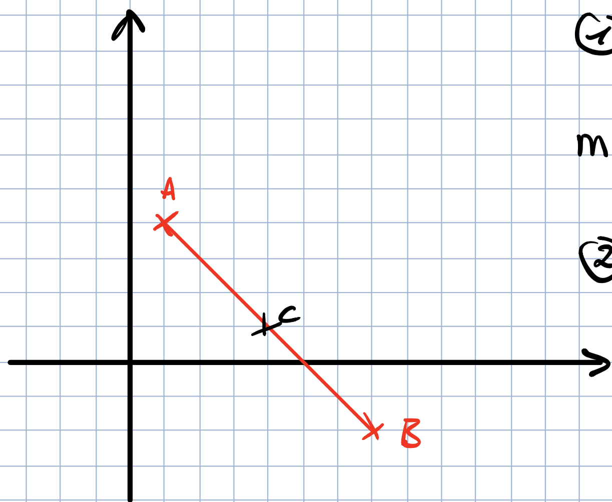
Exercise 5.
Determine the equation of the perpendicular bisector of the line segment between *A(1,4)* and *B(7,-2)*.



① Determine slope of AB

$$m_{AB} = \frac{-2-4}{7-1} = \frac{-6}{6} = -1$$

② Determine slope of line perpendicular to AB

$$\perp m_{AB} = \frac{-1}{m_{AB}}$$

$$\therefore \quad = \frac{-1}{-1} = 1$$

③ The mid-point (C) of line AB

$$C = \left(\frac{1+7}{2}, \frac{4-2}{2}\right) = (4,1)$$

④ Construct the perpendicular bisector

$$(y - y_1) = m(x - x_1)$$

$$(y-1) = 1\,(x-4)$$

$$y - 1 = x - 4$$

$$y = x - 3$$

Exercise 6.
Determine the equation of the tangent line to the circle $x^2 + y^2 = 25$ at the point *(3,-4)*. Give your answer in the form $ax + by + c = 0$.

① $y^2 = 25 - x^2$

$$2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = -x/y \qquad \frac{\text{slope at}}{(3,-4)} = \frac{-3}{-4} = 3/4$$

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$$(y-y_1) = m(x-x_1)$$
$$(y+4) = \tfrac{3}{4}(x-3)$$
$$y+4 = \tfrac{3}{4}x - \tfrac{9}{4}$$
$$y = \tfrac{3}{4}x - \tfrac{25}{4} \iff \tfrac{3}{4}x - y - \frac{25}{4} = 0$$

Exercise 7.
Draw the graph of the following functions. Determine any horizontal and/or vertical asymptote, and any intersection points of the graph with any of the two axes, the function might have.

a)

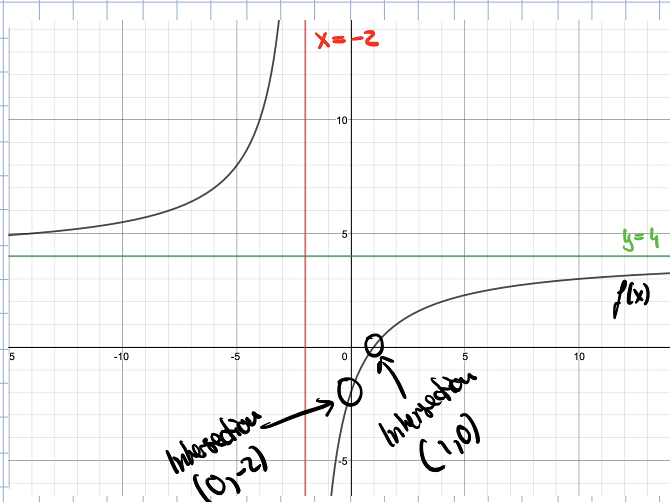
$$f(x) = \frac{4x-4}{x+2}$$

Vertical asymptote: $x+2=0 \rightarrow x=-2$

Horizontal asymptote: $\lim\limits_{x\to\infty} \frac{4x-4}{x+2} = \lim\limits_{x\to\infty} \frac{4x-4}{x+2} \left(\frac{1/x}{1/x}\right)$

$$= \lim_{x\to\infty} \frac{4-\frac{4}{x}}{1+\frac{2}{x}} \overset{\text{limit laws}}{=} \frac{\lim\limits_{x\to\infty} 4 - \lim\limits_{x\to\infty} \frac{4}{x}}{\lim\limits_{x\to\infty} 1 + \lim\limits_{x\to\infty} \frac{2}{x}}$$

$$= \frac{4}{1} = 4 = y$$



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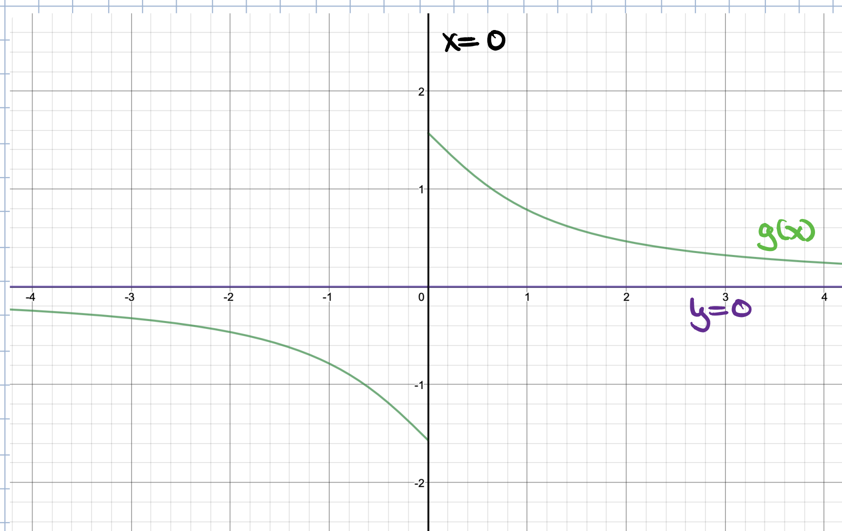
b) $g(x) = \arctan\left(\frac{1}{x}\right)$ (can also be denoted as $\tan^{-1}\left(\frac{1}{x}\right)$)

Vertical asymptote: $x = 0$

Horizontal asymptote: $\lim_{x\to\infty} \tan^{-1}\left(\frac{1}{x}\right) \overset{\text{limit law}}{=} \tan^{-1}\left(\lim_{x\to\infty}\frac{1}{x}\right)$

$= \tan^{-1}(0) = 0 = y$



c) $h(x) = \ln\left|1 + \frac{1}{x}\right|$

Vertical asymptote: $x = 0$

Horizontal asymptote: $\lim_{x\to\infty} \ln\left|1+\frac{1}{x}\right| \overset{\text{limit laws}}{=} \ln\left|\lim_{x\to\infty} 1 + \lim_{x\to\infty}\frac{1}{x}\right|$

$= \ln(1) = 0 = y$

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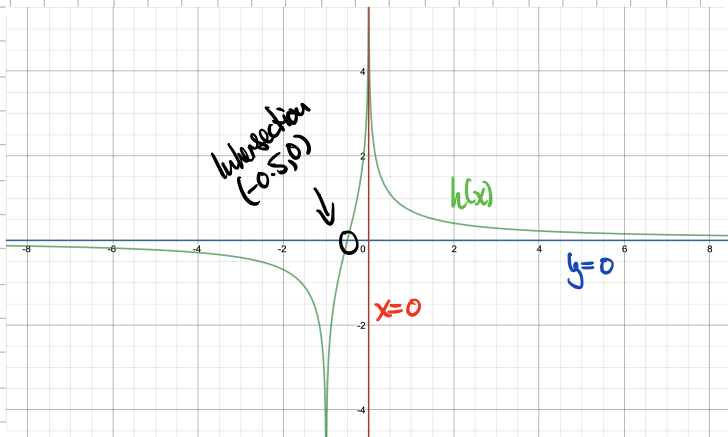
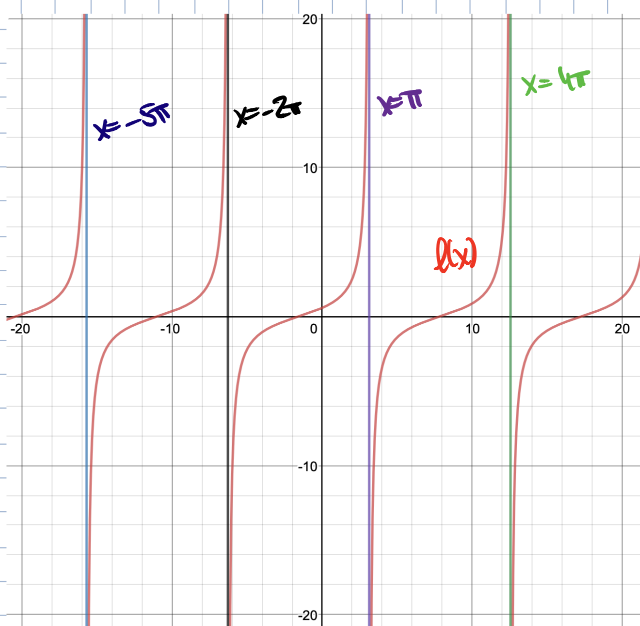


d) $l(x) = \tan\left(\frac{1}{3}x - \frac{17}{6}\pi\right)$

Vertical asymptote: tan(x) has vertical asymp. at $x = \pi + 3\pi n$

Horizontal asymptote: None



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Exercise 8.

Determine the equation $y = ax^2 + bx + c$ of the parabola (that is, determine *a*, *b* and *c*) with top *(0,0)* and which also goes through the point *(-1, –5)*.

① $ax^2+bx+c=y$ at $(0,0)$

$c=0$

② Maximum at $(0,0)$ therefore:

$$\frac{dy}{dx} = 2ax+b = 0 \quad \text{at } (0,0)$$

$b=0$

③ $ax^2+bx+c=y$, at $(-1,-5)$

$a-b+c=-5$, $c=0$

$a-b=-5$, $b=0$

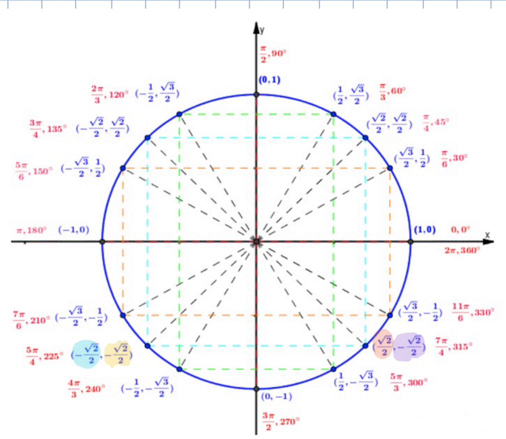
$a=-5$

$\therefore \quad y=-5x^2$

Exercise 9.

Given the angles $\alpha = \frac{5\pi}{4}$ and $\beta = \frac{7\pi}{4}$.

Draw a unit circle and determine the sign (positive or negative) of the following numbers: $\sin\alpha$, $\sin\beta$, $\cos\alpha$, $\cos\beta$, $\tan\alpha$, $\tan\beta$.



① $\sin\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

② $\sin\left(\frac{7\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

③ $\cos\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

④ $\cos\left(\frac{7\pi}{4}\right) = \frac{\sqrt{2}}{2}$

As $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$

⑤ $\tan\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2} \div -\frac{\sqrt{2}}{2} = 1$

⑥ $\tan\left(\frac{7\pi}{4}\right) = -\frac{\sqrt{2}}{2} \div \frac{\sqrt{2}}{2} = -1$

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Exercise 10.

Determine the derivative of the following functions:

a) $f(x) = \sqrt{x}\,\ln(\sin x)$

$f(x) = g(x)h(x)$

$f'(x) = g(x)h'(x) + h(x)g'(x)$

$g(x) = \sqrt{x}$ $\quad$ $h(x) = \ln(\sin(x))$

$g'(x) = \frac{1}{2}x^{-1/2}$ $\quad$ $h'(x) = \frac{1}{\sin(x)}\cos(x)$

$$f'(x) = \sqrt{x}\,\frac{\cos(x)}{\sin(x)} + \frac{\ln(\sin(x))}{2\sqrt{x}}$$

b) $g(x) = \frac{\sin x}{x+1}$

$g(x) = \frac{\ell(x)}{h(x)}$

$g'(x) = \frac{\ell'(x)h(x) - \ell(x)h'(x)}{(h(x))^2}$

$\ell(x) = \sin(x)$ $\quad$ $h(x) = x+1$

$\ell'(x) = \cos(x)$ $\quad$ $h'(x) = 1$

$$g'(x) = \frac{(x+1)\cos(x) - \sin(x)}{(x+1)^2}$$

c) $h(x) = e^{\arccos(x^2)}$ (that is, $h(x) = e^{\cos^{-1}(x^2)}$)

$\left.\begin{array}{l} \ell(x) = \cos^{-1}(x^2) \\ \ell'(x) = -\frac{2x}{\sqrt{1-x^4}} \end{array}\right\}$

$h(x) = e^{\ell(x)} = p(\ell(x))$

Where $p(x) = e^{\ell(x)}$

$h'(x) = p(\ell(x))\,\ell'(x)$

$$\therefore\ h'(x) = \frac{-2x e^{\cos^{-1}(x^2)}}{\sqrt{1-x^4}}$$

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d) $l(x) = \arctan(e^{-x})$.

$p(x) = \arctan(x) \rightarrow p'(x) = \dfrac{1}{x^2+1}$

$l(x) = p(e^{-x}) = p(h(x))$

$l'(x) = p(h(x))\, h'(x)$ where $h(x) = e^{-x}$

$= \dfrac{-e^{-x}}{e^{-2x}+1}$ $\qquad h'(x) = -e^{-x}$

Exercise 11.

Determine all local minima and/or maxima of the following functions and specify in each case whether you have found a minimum or a maximum.

a) $f(x) = e^{-|x|}$ $\quad x \in (-\infty, +\infty)$

① Differentiable on $x \in (-\infty, +\infty) \setminus \{0\}$

$f(x) = \begin{cases} e^{-x} & \text{for } x > 0 \\ e^{x} & \text{for } x < 0 \end{cases}$ $\quad f'(x) = -e^{-x}$, $f'(x) < 0$ for $\forall x \in (0, +\infty)$; $\quad f'(x) = e^{x}$, $f'(x) > 0$ for $\forall x \in (-\infty, 0)$

Monotonically increasing towards 1 and decreasing from 1, approaching $y = 0$ which is an asymptote, but no minimum

② Non-differentiable point $x = 0$

$f(0) = e^{-0} = 1$, as $f(0) > f(x)$ for $\forall x \in (-\infty, +\infty) \setminus \{0\}$

$x = 0$ is the maximum

b) $g(x) = (\ln x)^2$ $\quad x \in (0, +\infty)$

① $g(x) = (\ln(x))^2$ The function is differentiable on $x \in (0, +\infty)$

F.O.C. $\quad g'(x) = \dfrac{2(\ln(x))}{x}$

$\dfrac{2\ln(x)}{x} = 0 \rightarrow \ln(x) = 0 \rightarrow x = 1$

S.O.C

$g''(x) = -\dfrac{(2\ln(x) - 2)}{x^2}$

$g''(1) = -\dfrac{(2\ln(1) - 2)}{1^2} = 2$ $\qquad$ Thus we found a minimum

Solutions To Exercises On Entry Level (Instapniveau) For The Course Introduction To Analysis

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c) $h(x) = \arctan(x) \quad x \in (-\infty, +\infty)$

for $\lim\limits_{x \to +\infty} \arctan(x) = \frac{\pi}{2}$

for $\lim\limits_{x \to -\infty} \arctan(x) = -\frac{\pi}{2}$

two asymptotes at $y = \frac{\pi}{2}$ and $y = -\frac{\pi}{2}$ but not a minimum or maximum

d) $l(x) = \sqrt{x - 1} \quad x \in [1, +\infty)$

Differentiable on $(1, +\infty)$

$l(x) = (x-1)^{1/2}$

$l'(x) = \frac{1}{2}(x-1)^{-1/2} \Rightarrow \frac{1}{2}(x-1)^{-1/2} > 0$ for $\forall x \in (1, +\infty)$

hence the function is monotonically increasing, as such no maximum!

$\Rightarrow$ As monotonically increasing $x = 1$, must be the minimum as $f(1) < f(x)$ for $\forall x \in (1, +\infty)$

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Exercise 12.

Determine the following integrals:

a) $\int_e^{e^2} \frac{1}{x(1+\ln x)} dx.$

Integration by substitution, where $u = 1 + \ln(x)$

$$\int_2^3 \frac{1}{u}\, du = \ln|u| \Big|_2^3 = \ln(3) - \ln(2)$$

b) $\int_0^{\frac{\pi}{2}} \sin^3 x\, dx.$

$$\int_0^{\frac{\pi}{2}} \sin^2(x) \sin(x)\, dx = \int_0^{\frac{\pi}{2}} (1 - \cos^2(x)) \sin(x)\, dx$$

Use integration by substitution $u = \cos(x)$

$$\int_1^0 (u^2 - 1)\, du = \int_0^1 (1 - u^2)\, du = u - \frac{u^3}{3} \Big|_0^1$$

$$= \left(1 - \frac{1^3}{3}\right) - \left(0 - \frac{0^3}{3}\right) = \frac{2}{3}$$

c) $\int_0^1 \frac{-dx}{\sqrt{1-x^2}}.$

Standard integral:

$$\int \frac{1}{\sqrt{1-x^2}}\, dx = \sin^{-1}(x)$$

$$-\int_0^1 \frac{1}{\sqrt{1-x^2}}\, dx$$

$$= -\sin^{-1}(x) \Big|_0^1 = -\sin^{-1}(1) + \sin^{-1}(0) = -\frac{\pi}{2} + 0 = \frac{-\pi}{2}$$

d) $\int_0^1 \frac{dx}{x\sqrt{x}}.$

$$\int_0^1 \frac{1}{x^{3/2}} = \lim_{a \to 0} \int_a^1 x^{-3/2} = \lim_{a \to 0} -2x^{-1/2} \Big|_a^1 = -2 + \lim_{a \to 0} \frac{2}{\sqrt{a}}$$

$\Rightarrow$ The integral is divergent!

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