EXERCISES ON ENTRY LEVEL (INSTAPNIVEAU) FOR THE COURSE INTRODUCTION TO ANALYSIS

Do NOT use a calculator or any books when you set out to make the exercises!

- Give a valid argument for all steps you take.
- Simplify fractions as far as possible, but do write your final answer as a fraction, not as a decimal; do the same for square roots and logarithms (do not write them as decimals). Never use a calculator as it will also be prohibited to use one during the whole course, including the exam.

Exercise 1.
Solve the following inequalities and write the answer in one of the following forms: $a < x < b$, $a < x \leq b$, $a \leq x < b$, $a \leq x \leq b$.

a. $\frac{3}{x-1} - \frac{4}{x} \geq 1$.

b. $8 - |2x - 1| \geq 6$.

c. $\left|\frac{x-1}{x+1}\right| \leq 1$

Exercise 2.
Determine all possible values of x for which the following expressions will be real numbers:

a. $\left(\frac{1}{x^2-x-1}\right)^{\frac{1}{2}}$.

b. $\sqrt{x - \sqrt{|x-1|}}$.

c. $\ln(4x - |4x^2 - 1|)$.

Exercise 3.
For which real numbers x does the inequality $f(x) \geq g(x)$ hold? Given

$$f(x) = \text{x(x + 2)} \quad \text{and} \quad g(x) = 2x^2 + 4x.$$

Exercise 4.
Show that the points *A(1, 1)*, *B(7, 4)*, *C(5, 10)* and *D(-1, 7)* in the plane are in fact the vertices of a parallelogram, by purely making use of the slopes of the connecting lines between the vertices.

Exercise 5.
Determine the equation of the perpendicular bisector of the line segment between *A(1,4)* and *B(7,-2)*.

Exercise 6.
Determine the equation of the tangent line to the circle $x^2 + y^2 = 25$ at the point *(3,-4)*. Give your answer in the form $ax + by + c = 0$.

Exercise 7.
Draw the graph of the following functions. Determine any horizontal and/or vertical asymptote, and any intersection points of the graph with any of the two axes, the function might have.

a. $f(x) = \frac{4x-4}{x+2}$.
b. $g(x) = \arctan\left(\frac{1}{x}\right)$ (can also be denoted as $\tan^{-1}\left(\frac{1}{x}\right)$).
c. $h(x) = \ln\left|1 + \frac{1}{x}\right|$.
d. $l(x) = \tan\left(\frac{1}{3}x - \frac{17}{6}\pi\right)$.

Exercise 8.
Determine the equation $y = ax^2 + bx + c$ of the parabola (that is, determine *a, b* and *c*) with top *(0,0)* and which also goes through the point *(-1, –5)*.

Exercise 9.
Given the angles $\alpha = \frac{5\pi}{4}$ and $\beta = \frac{7\pi}{4}$.
Draw a unit circle and determine the sign (positive or negative) of the following numbers: $\sin\alpha$, $\sin\beta$, $\cos\alpha$, $\cos\beta$, $\tan\alpha$, $\tan\beta$.

Exercise 10.
Determine the derivative of the following functions:
a. $f(x) = \sqrt{x}\ \ln(\sin x)$.
b. $g(x) = \frac{\sin x}{x+1}$.
c. $h(x) = e^{\arccos(x^2)}$ (that is, $h(x) = e^{\cos^{-1}(x^2)}$).
d. $l(x) = \arctan(e^{-x})$.

Exercise 11.
Determine all local minima and/or maxima of the following functions and specify in each case whether you have found a minimum or a maximum.

a. $f(x) = e^{-|x|}$.
b. $g(x) = (\ln x)^2$.
c. $h(x) = \arctan(x)$.
d. $l(x) = \sqrt{x-1}$.

Exercise 12.
Determine the following integrals:

a. $\int_e^{e^2} \frac{1}{x(1+\ln x)}\,dx$.

b. $\int_0^{\frac{\pi}{2}} \sin^3 x\,dx$.

c. $\int_0^1 \frac{-dx}{\sqrt{1-x^2}}$.

d. $\int_0^1 \frac{dx}{x\sqrt{x}}$.