PRACTICE EXAM ANSWERS

Below, only the final answers are given. Note that at the actual entrance exam all necessary steps, formulas, substitutions, diagrams or graphs leading to your answer must be written down. Furthermore, questions containing the words “solve”, “derive” or ‘calculate” require an exact answer; a decimal approximation is not allowed.

**Question 1** maximum score: 4 points (2+2)

(a) \( x = -4 \) and \( y = 3 \)

(b) \( x = -3 \) and \( y = -4 \)

**Question 2** maximum score: 9 points (3+3+3)

(a) \( f'(x) = \frac{x}{\sqrt{x^2 + 4}} \)

(b) \( f'(x) = \frac{1}{x - 6} \)

(c) \( f'(x) = 1 \)

**Question 3** maximum score: 11 points (4+4+3)

(a) \( f'(x) = (-2x + 2)e^{-x^2 + 2x} = 0 \) when \( x = 1 \). Since \( f'(0) > 0 \) and \( f'(2) < 0 \), it follows that \( f \) has a maximum in \( x = 1 \)

(b) 2 inflection points: \( x = 1 \pm \frac{1}{2}\sqrt{2} \)

(c) Domain of \( f \) is equal to \( \mathbb{R} \)

**Question 4** maximum score: 9 points (3+3+3)

(a) \( x = 5 \) or \( x = -5 \)

(b) \( x = -1 \)

(c) \( x = -1 \). Note: the equation can be rewritten as \( 3^{2x} = 3^{-2(3x+4)} \)
Question 5 maximum score: 6 points (3+2+1)

(a) Note: the graph of \( f(x) \) is V-shaped and the graph of \( g(x) \) is a parabola
(b) \( x = 3 \) or \( x = -3 \)
(c) \( x \in (-\infty, -3] \cup [3, \infty) \)

Question 6 maximum score: 6 points (2+2+2)

(a) \( y = \frac{2}{5}x + 1 \)
(b) \( y = -\frac{5}{2}x + 28 \)
(c) \( y = 21x - 207 \)

Question 7 maximum score: 6 points (3+2+1)

(a) Note: the graph of \( f(x) \) is a straight line and the graph of \( g(x) \) consists of 2 separate parts
(b) \( x = 1\frac{1}{2} \) or \( x = -2\frac{1}{2} \)
(c) \( x \in [-2\frac{1}{2}, -\frac{1}{2}) \cup [1\frac{1}{2}, \infty) \)

Question 8 maximum score: 4 points (2+2)

(a) (i) Discriminant > 0 hence 2 solutions
    (ii) Discriminant < 0 hence no solutions
    (iii) Discriminant > 0 hence 2 solutions
(b) \( p \in (0, 12) \)

Question 9 maximum score: 5 points

Second point of inflection: \( (0, 40) \).
Note: \( a = 2 \) and \( b = 40 \)

Grade = Score/6