Please note:

- Write your student number on each sheet of the exam (this number is mentioned in the letter you have received from the exam administration).

- The use of a so called “graphing calculator” or “programmable calculator” is not permitted. “Simple” scientific calculators are allowed.

- Available time: 2.5 hours (150 minutes). You are not allowed to leave the examination room during the first hour of the exam and during the last 15 minutes.

- You CANNOT take any part of the exam home. All questions and answers have to be returned to the supervisor.

- In all your answers, give a complete solution where you show all the required steps, formulas, and substitutions that lead to your answer. A good or wrong answer is only a small part of the solution. The quality and completeness of your detailed solutions determine the points you will get. You should end an exercise with a conclusion or an answer.

- Questions containing the words “solve”, “derive” or “calculate” require an exact answer; a decimal approximation is not allowed.
Question 1

Solve each of the systems of equations below:

(a) \[
\begin{align*}
3x + 11y &= 21 \\
2x + 5y &= 7
\end{align*}
\]

(b) \[
\begin{align*}
5x - 2y &= -7 \\
11x - 3y &= -21
\end{align*}
\]

Question 2

Determine the derivative of each of the following functions, and simplify your answer as much as possible.

(a) \(f(x) = \sqrt{x^2 + 4}\)

(b) \(f(x) = \ln(x^2 - 6x) - \ln(x)\)

(c) \(f(x) = \frac{x^2 + 8x + 15}{5 + x}\)

Question 3

Given the function \(f(x) = e^{-x^2 + 2x}\)

(a) Determine the extreme values of \(f(x)\) or show that \(f\) has no extreme values. Determine for each extreme value whether it is a (local) maximum or a (local) minimum.

(b) Show that \(f''(x) = 2(2x^2 - 4x + 1)e^{-x^2 + 2x}\). Furthermore, determine the points of inflection of \(f\) or show that \(f\) has no inflection points.

(c) Determine the domain of function \(f\) and sketch the graph of \(f\), based on the answers to the previous questions.

Question 4

Solve the following equations:

(a) \(\ln(x^4 - 24x^2) - \ln(x^2) = 0\)

(b) \(\sqrt{x^2 - 15x - x} = 5\)

(c) \((3^x)^2 = \frac{1}{9^{3x+4}}\)
Question 5

Consider the functions \( f(x) = |2x| \) and \( g(x) = x^2 - 3 \)
Note: \(|x|\) denotes the absolute value of \(x\).
(a) Sketch the graphs of \(f\) and \(g\) in one figure.
(b) Solve \(|2x| = x^2 - 3\)
(c) Solve \(|2x| \leq x^2 - 3\)

Question 6

(a) Determine the formula of the straight line through the point \((10, 3)\), and parallel to the line \(y = \frac{2}{5}x + 13\).
(b) Determine the formula of the straight line through the point \((10, 3)\), and perpendicular to the line \(y = \frac{2}{5}x + 13\).
(c) Determine the formula of the tangent line of \(f(x) = \frac{2}{5}x^2 + 13x - 167\) at the point \((10, 3)\).

Question 7

(a) Sketch in the same figure the graphs of the functions
\[ f(x) = \frac{x + \frac{1}{2}}{2} \quad \text{and} \quad g(x) = \frac{2}{x + \frac{1}{2}} \]
(b) Calculate all points of intersection of the graphs of \(f\) and \(g\).
(c) Determine all the values of \(x\) for which the inequality \(f(x) \geq g(x)\) holds.

Question 8

(a) For each of the following three equations, give the number of solutions (explain your answer).
   (i) \(3x^2 - 2x - 2 = 0\)
   (ii) \(3x^2 + 8x + 8 = 0\)
   (iii) \(3x^2 + 20x + 20 = 0\)
(b) Give all value(s) of \(p\) for which the equation \(3x^2 + px + p = 0\) has no solutions.

Question 9

Consider the function \(f(x) = ax^4 - 8x^3 + b\). Assume that \((x, y) = (2, 8)\) is an inflection point of this function. Show that \(f\) has another inflection point and compute the \((x, y)\)-coordinates of this other point of inflection.