ERASMUS UNIVERSITY ROTTERDAM
Information concerning the Entrance examination Mathematics level 1
for International Bachelor in Communication and Media

General information

Available time: 2 hours 30 minutes. The examination consists of 15 questions. Only 8 out of the 15 questions have to be answered. Hence, in order to pass the exam, not every topic listed below has to be mastered. Each question is worth at most 3 points. In order to pass the examination a score of 12 points is sufficient.

In case a candidate answers more than the above mentioned number of questions, the points for the superfluously answered questions are not added to the total. In such cases, the examiner decides which of the answers will be regarded as superfluous.

All necessary steps, formulas, substitutions, diagrams or graphs leading to your final answer must be written down. Furthermore, questions containing the words "solve", "derive" or "calculate" require an exact answer; a decimal approximation is not allowed.

The use of a so called "graphing calculator" or "programmable calculator" is not permitted. "Simple" scientific calculators are allowed.

Content information

In the exam you may find questions regarding the following topics:

A: NUMBERS AND RULES OF CALCULATION

1. The sets $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$ and the operations addition, substraction, multiplication and division.

   Natural numbers: $\mathbb{N} = \{1, 2, 3, \ldots\}$
   Whole numbers: $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$
   Rational numbers: $\mathbb{Q} = \{\frac{a}{b} \mid a \in \mathbb{Z}, b \in \mathbb{N}\}$

   $a(b + c) = ab + ac$

   $(a + b)(c + d) = ac + ad + bc + bd$

   $(a + b)^2 = a^2 + 2ab + b^2$

   $(a - b)^2 = a^2 - 2ab + b^2$

   $a^2 - b^2 = (a + b)(a - b)$

   $$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$
   $$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$$
   $$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$
   $$\frac{a}{b} : \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

2. Absolute value $|x|$ and simple graphs corresponding with absolute value.

   $$|x| = \begin{cases} x & when \quad x \geq 0 \\ -x & when \quad x < 0 \end{cases}$$

3. The power rules and corresponding rules for logarithms. Use of rational and negative exponents.

   The next rules hold under the following conditions:
   $p, q \in \mathbb{Q}$, $a, b, g > 0$ *and* $g \in \mathbb{N}, g \neq 1$:

   $a^p \cdot a^q = a^{p+q}$
   $\frac{a^p}{a^q} = a^{p-q}$
   $(a^p)^q = a^{pq}$
   $(ab)^p = a^p b^p$
   $\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$
   $a^{-p} = \frac{1}{a^p}$
   $a^{\frac{1}{p}} = \sqrt[p]{a}$
   ${}^g\log a^p = p \cdot {}^g\log a$
   ${}^g\log a = \frac{\log a}{\log g} = \frac{\ln a}{\ln g}$
   ${}^g\log a + {}^g\log b = {}^g\log ab$
   ${}^g\log a - {}^g\log b = {}^g\log \frac{a}{b}$
   Remark: log *is the commonly used notation for* ${}^{10}\log$

B: STANDARD FUNCTIONS

1. Typical properties of the following standard functions:
   - polynomials,

     In particular:
     the linear function $y = ax + b$ *where* $x, a, b \in \mathbb{R}$
     the kwadratic function $y = ax^2 + bx + c$ *where* $x, a, b, c \in \mathbb{R}$

   - power functions $y = c \cdot x^n$, where $c \in \mathbb{R}$, $n \in \mathbb{Q}$,

     In particular the function $y = \sqrt{x}$ *with* $x \geq 0$

   - exponential functions $y = g^x$ where $g \in \mathbb{N}$ and their inverse functions $y = {}^g\log x$, where $g \in \mathbb{N}, g \neq 1$ and $x > 0$,
   - the function $y = e^x$, where $x \in \mathbb{R}$ and its inverse function $y = \ln x$, for $x > 0$,
   - the functions $y = \sin x$ and $y = \cos x$, where $x \in \mathbb{R}$

     In particular the following properties:
     $-\sin x = \sin(-x) = \sin(\pi + x)$
     $-\cos x = \cos x = \cos(\pi + x) = \cos(\pi - x)$
     $\sin x = \cos(\frac{\pi}{2} - x)$
     $\cos x = \sin(\frac{\pi}{2} - x)$
     $\sin^2 x + \cos^2 x = 1$

2. Use combinations and inverses of the standard functions.

   For example: $y = \sqrt{3x - 6}$ *where* $x \geq 2$.

3. Sketch the graphs of the standard functions and use the notions domain (all possible values of x for which the function is defined), range (the set of all values of y obtained as x varies in the domain), zeroes, ascending and descending function.
4. Sketch the graphs of the goniometric functions $y = \sin x$ and $y = \cos x$, using the notions period ($= 2\pi$), amplitude ($= 1$) and equilibrium value ($= 0$).
5. Apply transformations on graphs such as shifting and stretching, and describe the link between the transformation and the alteration of the corresponding function.

   For example, given a function $f(x)$:
   - *translation of $f(x)$ along the vector (a, b) results in $y = f(x - a) + b$*
   - *multiplication of $f(x)$ with a with respect to the x-axis results in $y = a \cdot f(x)$*
   - *multiplication of $f(x)$ with b with respect to the y-axis results in $y = f(x/b)$*

C: EQUATIONS AND INEQUALITIES

1. Factorization of quadratic expressions into linear terms.

   For example:
   - *$f(x) = x^2 - 2x - 15$ can be factorized as follows: $f(x) = (x - 5)(x + 3)$*
   - *$f(x) = x^2 - 2x$ can be factorized as follows: $f(x) = x(x - 2)$*

2. Compute the solution of a quadratic equation, using factorization or the following formula:

   $ax^2 + bx + c = 0$ *has solutions* $x_{1,2} = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

3. Handle the following equalities and their solutions:

$$\begin{array}{lll} x^a = b & \Rightarrow & x = b^{\frac{1}{a}} = \sqrt[a]{b} \\ g^x = a & \Rightarrow & x = {}^g\log a = \frac{\log a}{\log g} = \frac{\ln a}{\ln g} \\ {}^g\log x = a & \Rightarrow & x = g^a \end{array}$$

4. The solution of $\sin x = 0$, $\cos x = 0$, $\sin t = \sin u$, and $\cos t = \cos u$:

$$\begin{array}{llll} \sin x = 0 & \Rightarrow & x = k\pi & k \in \mathbb{Z} \\ \cos x = 0 & \Rightarrow & x = \pi/2 + k\pi & k \in \mathbb{Z} \\ \sin t = \sin u & \Rightarrow & t = u + 2k\pi \text{ or } t = \pi - u + 2k\pi & k \in \mathbb{Z} \\ \cos t = \cos u & \Rightarrow & t = u + 2k\pi \text{ or } t = -u + 2k\pi & k \in \mathbb{Z} \end{array}$$

5. Solve inequalities using a sketch.

   For example: $x^2 - 3x - 10 > -x + 5$.
   Solving the equation gives: $x = 5$ or $x = -3$.
   A sketch of the graphs supplies the answer to the problem: $x < -3$ or $x > 5$.

D: CALCULUS

1. Recognition and use of the different symbols for the derivative of a function.

   $f'(x)$, $\frac{dy}{dx}$, $\frac{df(x)}{dx}$ and $\frac{df}{dx}(x)$ all denote the same notion.

2. Compute the derivative of the sum, the product, the quotient and (in simple cases) of combinations of standard functions, as described in items B1 and B2. Compute the derivative of a sum, product, or quotient of functions. Use the chain rule.

   Some examples:

sum:	$f(x) = x^2 + \sqrt{x}$	$\Rightarrow$	$f'(x) = 2x + \frac{1}{2\sqrt{x}}$
product:	$f(x) = (x^2 - 3x + 5)\ln x$	$\Rightarrow$	$f'(x) = (2x - 3)\ln x + (x^2 - 3x + 5) \cdot \frac{1}{x}$
quotient:	$f(x) = \frac{x^2}{5x+3}$	$\Rightarrow$	$f'(x) = \frac{(5x+3)\cdot 2x - x^2(5)}{(5x+3)^2} = \frac{x^2+6x}{(5x+3)^2}$
chain rule:	$f(x) = (5x^2 + 7)^4$	$\Rightarrow$	$f'(x) = 4(5x^2 + 7)^3 \cdot 10x = 40x(5x^2 + 7)^3$

3. Determine on which intervals a function is constant (derivative $= 0$), ascending (derivative > 0) or descending (derivative < 0).

E: STRAIGHT LINES AND SYSTEMS OF EQUATIONS

1. Determine the formula of a straight line either in case two of its points are given, or in case one single point is given, combined with the slope. (Also, see D5.)
2. Know the conditions for two parallel straight lines and two perpendicular straight lines. (Also, see D5)

   The two lines $y = ax + b$ and $y = cx + d$ are parallel when $a = c$. The lines are perpendicular when $ac = -1$.

3. Solve a system of two linear equations with two unknowns.

   For example:

$$\begin{cases} 2x + 2y = 2 \\ 3x + y = 5 \end{cases} \Rightarrow \begin{cases} x = 2 \\ y = -1 \end{cases}$$

4. Sketch the solution of a system with linear inequalities.

F: COMBINATORICS

1. In connection with a given combinatorics problem, draw a suitable visualization, e.g. a tree diagram, a network or a grid.

   For example: when you are throwing with two dice, a grid is very useful to arrange the outcomes.

2. Use permutations in case the order of the chosen items is essential.
   The number of permutations of r items out of a set of n is equal to

$$r \text{ out of } n = P_k^n = \frac{n!}{(n-r)!}$$

   For example: choosing a chair-man, a secretary and a treasurer out of a group of 5 persons, can be done in 60 different ways.

3. Use combinations in case the order of the chosen items is not important.
   The number of combinations of r items out of a set of n is equal to

$$n \text{ above } r = C_k^n = \left(\begin{array}{c} n \\ r \end{array} \right) = \frac{n!}{(n-r)!r!}$$

   For example: choosing a committee of 3 persons out of a group of 5 persons, can be done in 10 different ways.

4. Given a combinatorics problem, determine whether the problem indicates that repetition is allowed or not.

   For example: when choosing three letters from the alphabet, allowing repetition yields $26 \cdot 26 \cdot 26$ *different possibilities. Choosing three letters without repetition yields* $26 \cdot 25 \cdot 24$ *possibilities.*

5. Determine whether the multiplication rule for independent occasions can be applied. (Also see G5 and G7.)

   Consider the example above: when repetition is allowed, the drawings of the letters are independent. When repetition is not allowed, the outcome of the second and third drawing is depending on the first outcome. So in this case the three drawings are not independent.

G: PROBABILITY

1. At experiments dealing with probabilities, use the notions outcomes, set of outcomes, event, elementary event, impossible event and contradictory events.
2. Translate experiments regarding probability to a model with a jar containing marbles. Determine whether the order in which the marbles are drawn is important or not, and determine whether, after each drawing, the marble has to be put back into the jar or not.

   - *When after a drawing the possibilities remain the same, this is modelled by a jar of marbles where the marbles have to be put back into the jar after each drawing.*
   - *Simultaneous drawings translate to a model with a jar of marbles where two marbles are being drawn without putting the first one back.*

3. Calculate probabilities by using symmetry and by counting systematically.
4. Recognize combinatoric aspects when counting the number of elements of a set of outcomes. Use of the following rule:
   Probability = (number of favorable outcomes) / (total number of outcomes)

5. Calculate probabilities by using the multiplication rule for independent occasions. (Also, see F5 and G7.)

   - *Multiplication rule: for independent events A and B it holds:*

   $$P(A \text{ and } B) = P(A) \cdot P(B)$$

   Counterexample: throwing two dice; X represents the total number thrown. $P(X > 9 \text{ and } X \text{is even}) = \frac{4}{36} \neq P(X > 9) \cdot P(X \text{is even}) = \frac{1}{6} \cdot \frac{1}{2}$.

6. Calculate probabilities by using the sum rule or the complement of a probability.

   - *Sum rule: when events A and B rule each other out:* $P(A \text{ or } B) = P(A) + P(B)$.
   - *Using the complement:* $P(A) = 1 - P(\text{non-}A)$.

7. Use the notions of independent events and conditional probability when dealing with symmetrical and non-symmetrical probability spaces. (Also, see F5 and G5.)

   - *The events A and B are independent when*

   $$P(A \text{ and } B) = P(A) \cdot P(B)$$

   - *A conditional probability satisfies* $P(A \mid B) = P(A \text{ and } B)/P(B)$.

8. Describe the set of values for a discrete probability variable (in simple cases, with the probability distribution).

   For example: throwing four dice. We are interested in the number of dice that show six dots. We denote the probability value by X, hence X = the number of dice that show six dots. X can attain the values $0, 1, 2, 3$ *or* 4.
   - $P(X = 0) = \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} = \frac{625}{1296}$
   - $P(X = 1) = 4 \cdot \frac{1}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} = \frac{500}{1296}$
   - $P(X = 2) = 6 \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} = \frac{150}{1296}$
   - $P(X = 3) = 4 \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{5}{6} = \frac{20}{1296}$
   - $P(X = 4) = \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{1296}$

   Note that $P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) = 1$.

9. Calculate and interpret the expected value of a discrete probability variable when its distribution of probabilities is given.

   In the above example, the expected value $E(X)$ *of the discrete probability variable X is equal to*

   $$E(X) = 0 \cdot \frac{625}{1296} + 1 \cdot \frac{500}{1296} + 2 \cdot \frac{150}{1296} + 3 \cdot \frac{20}{1296} + 4 \cdot \frac{1}{1296} = \frac{864}{1296} = \frac{2}{3}$$

10. Apply the following rule: "The expected value of a sum = the sum van the expected values".

H: REMAINING TOPICS

1. Use the formula $N(t) = N_0 \cdot g^t$, which denotes exponential growth.

   Example: A bacterial population (starting with $100\,000$ *bacteria) grows exponentially with the hourly factor of growth equal to* $1,5$ *(so, every hour 50% of the present number of bacteria is added). This population is described by the formula* $N(t) = 100\,000(1,5)^t$.

2. Convert factors of growth and percentages of growth.

   Example: an hourly factor of growth equal to $1,5$ *converts to a factor of growth equal to* $(1,5)^{\frac{1}{4}} \approx 1,1067$ *for every quarter of an hour. Hence, an hourly percentage of growth equal to* 50% *converts to a growth of* $10,67\%$ *per quarter of an hour.*

3. Make calculations involving "compound interest"

   Example: in 10 years, a capital of $100\,000$ *euro, that is deposited at an annual interest rate of* 5% *will have grown to* $100\,000(1,05)^{10}$ *euro.*

4. Calculate the so-called "measures of center": mean, median and mode.

   The mean (=average) of the values $1, 1, 1, 2, 2, 4, 4, 7, 9,$ *and* 9 *is equal to* $\frac{40}{10} = 4$. *The median (the middle value, or the mean of the middle two values, when the data is arranged in numerical order) is equal to* $\frac{2+4}{2} = 3$, *and the mode (the value that appears the most) is equal to* 1.

5. Given a set of data, create a frequency table, in some cases using class (or group) intervals.